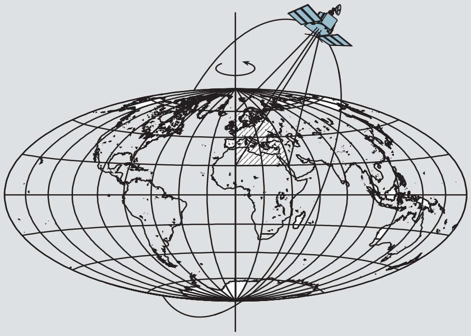
Applications of Parameter Estimation and Hypothesis Testing to GPS Network Adjustments

by

Kyle Brian Snow



Report No. 465

Geodetic and GeoInformation Science
Department of Civil and Environmental Engineering and Geodetic Science
The Ohio State University
Columbus, Ohio 43210-1275

December 2002

APPLICATIONS OF PARAMETER ESTIMATION
AND HYPOTHESIS TESTING
TO GPS NETWORK ADJUSTMENTS

by

Kyle B. Snow

Report No. 465

Department of Civil and Environmental Engineering and Geodetic Science
The Ohio State University
Columbus, Ohio 43210

December, 2002

ABSTRACT

It is common in geodetic and surveying network adjustments to treat the rank deficient normal equations in a way that produces zero variances for the so–called "control" points. This is often done by placing constraints on a minimum number of the unknown parameters, typically by assigning a zero variance to the a priori values of these parameters (coordinates). This approach may require the geodetic engineer or analyst to make an arbitrary decision about which parameters to constrain, which may have undesirable effects, such as parameter error ellipses that grow with distance from the constrained point.

Constraining parameters to a priori values is only one way of overcoming the rank deficiency inherent in geodetic and surveying networks. There are more preferable ways, which this thesis presents, namely Minimum Norm Least–Squares Solution (MINOLESS) and Best Linear Minimum Partial Bias Estimation (BLIMPBE). MINOLESS not only minimizes the weighted norm of the observation error vector but also minimizes the norm of the parameter vector, while BLIMPBE minimizes the bias for a subset of the parameters. In this thesis, these techniques are applied to a geodetic network that serves as a datum access for GPS–buoy work in Lake Michigan. The GPS–buoy has been used extensively in recent years by NOAA, The Ohio State University (OSU), and other organizations to determine lake and ocean surface heights for marine navigation and scientific studies. The work presented in this paper includes 1) parameter estimation using (Weighted) MINOLESS and hypothesis testing for the purpose of determining if recent observations are consistent with published coordinates at an earlier epoch; 2) a discussion of the BLIMPBE estimation technique for three new points to be used as GPS–buoy fiducial stations and a comparison of this technique to the "Adjustment with Stochastic Constraints" method; 3) usage of standardized reliability numbers for correlated observations; 4) a proposal for outlier detection and minimum outlier computation at the GPS–baseline level. The work may also be used as an example to follow for establishing new fiducial points with respect to a geodetic reference frame using observed GPS baseline vectors.

The results of this work lead to the following conclusions: 1) MINOLESS is the parameter estimation techniques of choice when it is required that changes to all a priori coordinates be minimized while performing a minimally constrained adjustment; 2) BLIMPBE appears to be an attractive alternative for selecting subsets of the parameter vector to adjust. BLIMPBE solutions using various selection–matrix types are worthy of further investigation; 3) outlier detection at the GPS–baseline level permits the entire observed baseline to be evaluated at once, rather than making decisions regarding the hypothesis at the baseline–component level. It is shown that the two approaches can yield different results.

Dedicated to my wife Karla and daughters Kyla and Kate

PREFACE

This report was prepared by Kyle Snow while a student in the Department of Civil and Environmental Engineering and Geodetic Science. It was submitted to the Graduate School of The Ohio State University in the Autumn of 2002 in partial fulfillment of the requirements of the Master of Science degree. Prof. Burkhard Schaffrin served as advisor and Prof. C.K. Shum as co–advisor, both in the program for Geodetic Science and Surveying. The research was funded in part by the Office of Naval Research Naval Oceanographic Partnership Program (NOPP), under the Ohio State University component of the Gulf of Mexico Monitoring System, and the NASA Physical Oceanography program under the TOPEX/POSEIDON Extended Mission project.

ACKNOWLEDGMENTS

I would like to thank Mr. Robert J. Bittel, licensed surveyor in the state of California, for teaching me the fundamentals of field surveying and for presenting me with a business opportunity, the earnings from which made it possible for me to return to full time academic study.

I would also like to thank Drs. Musthaq Hussain and Fareed Nader of the California State University, Fresno. Their respective courses in geodesy and satellite geodesy were taught with such excellence and made so interesting as to inspire me to study geodesy at the graduate level.

I am grateful to The Ohio State University for providing me with research and teaching associate positions, without which the move to Ohio for fulltime studies would not have been possible. In particular, I am grateful to my co–advisor Dr. C.K. Shum for providing the research funds to conduct the field campaign for collecting the data used in this study. These funds were supplied in part by the Office of Naval Research Naval Oceanographic Partnership Program (NOPP), under the Ohio State University component of the Gulf of Mexico Monitoring System, and the NASA Physical Oceanography program under the TOPEX/POSEIDON Extended Mission project.

I am also very grateful for the opportunity to be the student of my advisor Dr. Burkhard Schaffrin, and especially for the opportunity to take advanced courses from Dr. Schaffrin, where the theories behind the discussions of this thesis were learned. I acknowledge Mr. Steve Hilla, M.Sc., of the NGS for his time in working with me with PAGES software. Finally, I thank Mr. Douglas Bruce for pointing out several grammatical errors and inconsistencies in the final draft. Any remaining errors in the text are solely my responsibility.

TABLE OF CONTENTS

Chapters:

Appendices

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

BIQUUE	Best Invariant Quadratic Uniformly Unbiased Estimation
BLIMPBE	Best LInear Minimum Partial Bias Estimation
BLUMBE	Best Linear Uniformly Minimum Bias Estimation
BLUP	Best Linear Unbiased Prediction
BLUUE	Best Linear Uniformly Unbiased Estimation
CORS	Continuous Operating Reference Station
DOY	Day Of Year
GMM	Gauss–Markov Model
GPS	Global Positioning System
IGS	International GPS Service
LESS	LEast–Squares Solution
LHS	Left Hand Side
IERS	International Earth Rotation Service
ITRF	International Terrestrial Reference Frame
ITRS	International Terrestrial Reference System
MINOLESS	MInimum NOrm LEast–Squares Solution
NAD83	North American Datum of 1983
NAVD 88	North American Vertical Datum of 1988
NIMA	National Imagery and Mapping Agency
NGS	National Geodetic Survey
NOAA	National Oceanic and Atmospheric Administration
OSU	The Ohio State University
PAGES	Program for Adjustment of GPS Ephemerides
RHS	Right Hand Side
RLESS	Restricted LEast–Squares Solution
RMS	Root Mean Square
SCLESS	Stochastically Constrained LESS
WMINOLESS	Weighted MINOLESS

CHAPTER 1

INTRODUCTION

The primary objective of this study is to estimate the coordinates of three stations along the shore of Lake Michigan that are intended to be used as GPS–buoy fiducial stations (hereinafter referred to as fiducial stations). The chief interest is in the estimated ellipsoidal heights of the new fiducial stations. The survey method is static GPS with dual–frequency phase observables. The geodetic reference system is the ITRS96 (International Terrestrial Reference System – 1996), which is realized through the ITRF96 (International Terrestrial Reference Frame – 1996). Though this work was done in support of concurrent GPS–buoy data collection (Cheng et al., 2001), it is also offered as an example of how fiducial point coordinates can be established for future GPS–buoy projects. Different least–squares techniques for estimating parameters will be presented and compared. Standardized reliability numbers for correlated observations and an approach for detecting outliers in observed baseline vectors at the baseline level are also presented.

In the United States, the network of GPS Continuously Operating Reference Stations (CORS) managed by the National Geodetic Survey (NGS) provides the best local access to the ITRF96, indirectly through nearly 200 CORS and directly through nine of these that are also International GPS Service (IGS) stations.[1] Data from the IGS stations are used by the International Earth Rotation Service (IERS) in the computation of the ITRF.[2] The NGS uses a minimum of ten days of 24–hour observation sessions, but typically many more, to estimate the coordinates and velocity vectors of the CORS with respect to the ITRF.[3] The NGS publishes geodetic and Cartesian coordinates and velocity vectors in both the ITRF and the North American Datum of 1983 (NAD83) systems (see data sheets in Appendix A). The estimates are with respect to epoch 1997.0, which is the official epoch of these systems (NIMA, 2000). Dispersions of the estimated coordinates (parameters) are not published by the NGS. However, the author has learned through correspondence with NGS personnel that the nominal standard deviations of the coordinates are considered to be ±1 cm in the horizontal components and ±2 cm in the vertical direction; these values are considered to be at the 2–sigma confidence level.[4] Although the NGS estimates both the horizontal– and vertical–velocity vector components, due to “the fact that CORS data span too short of a time period to provide statistically meaningful vertical velocities,” the vertical velocity is listed as zero (see data sheets in Appendix A).[5] Only those stations included in the ITRF have published vertical velocities, which were estimated by the IERS (see NLIB data sheet in Appendix A).

As noted above, the estimated heights of the new stations are of primary interest. Without the means to project the published heights (1997.0 epoch) to the project epoch (June 1999) through a known velocity vector, one may question whether the published heights of the CORS stations represent a homogeneous data set at the time of the field campaign. Therefore, *the first task is to validate the published height values* through new estimates and subsequent hypothesis testing to see if the published values agree with current observational data. The *second task described in this paper is the coordinate estimation of the three new fiducial points*. These two tasks are treated individually and are presented in Chapters 4 and 5 respectively.

Before beginning with either of the above mentioned tasks, all formulae used in this thesis are presented in Chapter 2.

CHAPTER 2

MATHEMATICAL MODELS FOR ADJUSTMENTS AND HYPOTHESIS TESTING

The fundamental Gauss–Markov Model (GMM) is presented first in this chapter, followed by the equations and solutions for all the particular adjustment models used herein. All models are given in their linear form. In general, the development of each adjustment solution begins with a Lagrange target function to be minimized using the techniques of calculus. Typically, the weighted norm of the predicted error vector is minimized under certain prescribed conditions. Statistical and geometric properties of the adjustment solutions are mentioned briefly. Finally, equations used for outlier detection and hypothesis testing are shown.

The following comments are made about the symbolic notation used in this text. Lowercase Greek letters are used for nonrandom variables only. Lowercase letters are used for scalars and column vectors while uppercase letters are reserved for matrices. Whether a variable (or the digit 0) represents a scalar or vector should be clear from the context. Estimated nonrandom variables have hats on top, and tildes are used to denote predicted random variables. The definition of all variables used throughout the paper will be given in Chapter 2. The symbol $\hat{\xi}$ is sometimes used with a subscripted name to denote the type of solution it represents. When no subscript is shown, the type of solution is assumed to be clear from the context. The following symbols are also used: $\mathcal{R}(\cdot)$ denotes the range (column) space of its argument; $\mathrm{rk}(\cdot)$ means the rank of the matrix; $\mathrm{tr}(\cdot)$ is used for the trace of a matrix; $\mathbb{R}^m$ denotes the *m*–dimensional field of real numbers; $\oplus$ and $\overset{\perp}{\oplus}$ are used for the direct sum and complementary (orthogonal) sum, respectively, of two column spaces.

2.1 Least–Squares Adjustment Models

From the models given in each section, the *LEast–Squares Solutions* (LESS) are developed or given, and formulae for the parameter dispersions are shown. Important characteristics of the model, such as the rank of the normal matrix, the constraints imposed, or the bias properties of the solution, are typically noted. Frequent references are made to the literature where these characteristics are discussed in greater detail.

2.1.1 Gauss–Markov Model, LESS, and BLUUE

The Gauss–Markov Model (GMM) expresses the vector of observations as a function of the parameters and states the random nature of the observation errors. The linearized form of the model is

$$\underset{n\times 1}{y} = \underset{n\times m}{A}\,\xi + e, \quad e \sim \left(0, \sigma_0^2 P^{-1}\right), \quad \operatorname{rk}\left(A\right) =: q \le \left\{m, n\right\}. \tag{1}$$

This is the general case, where A may or may not be of full column rank. Because of linearization, y is the vector of n observations minus the zero–order terms, A is the (known) $n\times m$ coefficient matrix containing first–order derivatives of the observations with respect to the m unknown parameters, ξ is the parameter vector to estimate (corrections to a priori coordinates), and e is the vector of observation errors that are considered to be random and have zero expectation. The $n\times n$ matrix P contains weights of the observations, which may be correlated. The inverse of P shown in (1) implies that P is a positive definite matrix; this inverse matrix is called the cofactor matrix and is often denoted by Q in the literature. The symbol σ_0^2 is the a priori reference variance, which can also be estimated. The letter q denotes the rank of matrix A. The redundancy of the system of equations in (1) is defined as

$$r := n - \operatorname{rk}(A) = n - q\,. \tag{2}$$

A least–squares solution of (1) can be derived by minimizing the quadratic form $e^{\mathrm{T}}Pe$ while simultaneously satisfying the relation between the errors and observations expressed in (1). This leads to the following Lagrange target function to be minimized:

$$\Phi\left(e,\xi,\lambda\right) = e^{\mathrm{T}}Pe + 2\lambda^{\mathrm{T}}\left(y - A\xi - e\right) = \underset{\left(e,\xi,\lambda\right)}{\text{stationary}}\,. \tag{3}$$

Here, λ is a $n\times 1$ vector of Lagrange multipliers. The term “stationary” over the variables denotes that point in the domain of the function where $\Phi\left(e,\xi,\lambda\right)$ becomes stationary, i.e., where the derivative of the function is zero (global minimum sought in this case). The Euler–Lagrange necessary conditions are formed by setting the partial derivatives of (3) equal to zero as follows:

$$\begin{aligned}
\frac{1}{2}\frac{\partial\Phi}{\partial e} &= P\tilde{e} - \hat{\lambda} \doteq 0\\
\frac{1}{2}\frac{\partial\Phi}{\partial \xi} &= -A^{\mathrm{T}}\hat{\lambda} \doteq 0\\
\frac{1}{2}\frac{\partial\Phi}{\partial \lambda} &= y - A\hat{\xi} - \tilde{e} \doteq 0.
\end{aligned} \tag{4}$$

The hat symbols now denote particular vectors, i.e., solutions to the homogeneous system of equations. The second partial–derivative of Φ with respect to e yields the positive definite P matrix, which satisfies the sufficient conditions of the minimization problem. After algebraic manipulation of (4), the following normal equations can be written

$$N\hat{\xi} = c, \text{ with } \begin{bmatrix} \mathrm{N}, & c \end{bmatrix} = A^{\mathrm{T}} P \begin{bmatrix} A, & y \end{bmatrix}. \tag{5}$$

From (5), any LESS with its dispersion matrix (by variance propagation) is represented by

$$\hat{\xi} = N^{-} c \tag{6a}$$

$$D\{\hat{\xi}\} = \sigma_0^2 N^{-} N \left(N^{-}\right)^{\mathrm{T}} = \sigma_0^2 N_{rs}^{-}. \tag{6b}$$

The corresponding predicted error vector and its associated dispersion matrix are

$$\tilde{e} = y - A\hat{\xi} = \left(I_n - AN^{-}A^{\mathrm{T}}P\right) y \tag{7a}$$

$$D\{\tilde{e}\} = \sigma_0^2 \left(P^{-1} - AN^{-}A^{\mathrm{T}}\right) = D\{y\} - D\{A\hat{\xi}\} = \sigma_0^2 Q_{\tilde{e}}. \tag{7b}$$

Equations (7a) and (7b) for the predicted error vector and its dispersion are computed the same way for all the models presented herein unless noted otherwise, with the appropriate substitution for $\hat{\xi}$ and N^{-}, respectively. The symbol $Q_{\tilde{e}}$ denotes the cofactor matrix of $\tilde{e}$. The symbol N^{-} represents a generalized inverse of N. The generalized inverse is not unique; it is only required that it satisfies the definition of a generalized inverse: $NN^{-}N = N$. It can be shown that the matrix product in (6b) is a symmetrical reflexive generalized inverse (N_{rs}^{-}) of N. [6] Such a generalized inverse has the properties: $NN_{rs}^{-}N = N$, $N_{rs}^{-}NN_{rs}^{-} = N_{rs}^{-}$, implying $\mathrm{rk}\left(N_{rs}^{-}\right) = q$ and $N_{rs}^{-} = \left(N_{rs}^{-}\right)^{\mathrm{T}}$. Therefore, any solution of (6a) can be represented by $\hat{\xi} = N_{rs}^{-}c$ if the dispersion matrix becomes $D\{\hat{\xi}\} = \sigma_0^2 N_{rs}^{-}$.

If A were of full column rank, then the equation $N^{-} = N^{-1}$ would hold. Under such a condition, the LESS is a *Best Linear Uniformly Unbiased Estimate* (BLUUE) of ξ (SCHAFFRIN, 1997), where "Best" is used in the sense of a minimum trace of the dispersion matrix, and "Uniformly Unbiased" means the solution is unbiased for all $\xi \in \mathbb{R}^m$. But since (1) does not necessarily require that A be full column rank, and since A and N are of the same rank (KOCH, 1999, pg. 20), equation (5) cannot be uniquely solved without additional a priori information (i.e., some minimum constraint associated with ξ).

The potential rank deficiency of A is also referred to as “datum deficiency,” which gets its name from the geometric quantities comprising a geodetic or surveying network (in a three–dimensional network: scale, three rotations, and three orientations). Thus, treating the rank deficiency is also referred to as “defining the datum.” The following sections discuss various methods for handling rank deficiency in a geodetic network.

2.1.2 RLESS

Oftentimes, the minimally constrained solution for LESS is computed by the technique of *Restricted LESS* (RLESS). The development of RLESS is based upon the constraint equation $K\xi = \kappa_0$, which imposes a minimum number of constraints and thus removes the datum deficiency inherent in (1). In order to have a set of minimum constraints, the $l \times m$ matrix K must satisfy the following conditions.

$$\mathcal{R}\left(K^{\mathrm{T}}\right) \cap \mathcal{R}\left(A^{\mathrm{T}}\right) = \{0\} \text{ and } \mathcal{R}\left(K^{\mathrm{T}}\right) \cup \mathcal{R}\left(A^{\mathrm{T}}\right) = \mathbb{R}^m \Leftrightarrow \tag{8a}$$

$$\mathcal{R}\left(K^{\mathrm{T}}\right) \oplus \mathcal{R}\left(A^{\mathrm{T}}\right) = \mathbb{R}^m \Leftrightarrow \tag{8b}$$

$$m = r\mathrm{k}\left[A^{\mathrm{T}}, K^{\mathrm{T}}\right] = \mathrm{rk}\left(A\right) + \mathrm{rk}\left(K\right) = q + \left(m - q\right) \Rightarrow \tag{8c}$$

$$\mathrm{rk}\left(K\right) =: l = m - q \tag{8d}$$

Given these properties for K, the following Lagrange target function is minimized:

$$\Phi\left(\xi, \lambda\right) = e^{\mathrm{T}} P e + 2\lambda^{\mathrm{T}}\left(K\xi - \kappa_0\right) = \underset{(\xi,\lambda)}{\text{stationary}} . \tag{9}$$

Again, λ is a vector of Lagrange multipliers. The Euler–Lagrange necessary conditions are formed by setting the partial derivatives of (9) equal to zero as follows:

$$\begin{aligned} \frac{1}{2}\frac{\partial \Phi}{\partial \xi} &= N\hat{\xi} - c + K^{\mathrm{T}}\hat{\lambda} \doteq 0 \\ \frac{1}{2}\frac{\partial \Phi}{\partial \lambda} &= K\hat{\xi} - \kappa_0 \doteq 0. \end{aligned} \tag{10}$$

The sufficient condition is confirmed by $\frac{1}{2}\frac{\partial^2 \Phi}{\partial \xi \partial \xi^{\mathrm{T}}} = N$, which is positive (semi) definite. Equation (10) can be written in matrix form as

$$\begin{bmatrix} N & K^{\mathrm{T}} \\ K & 0 \end{bmatrix} \begin{bmatrix} \hat{\xi} \\ \hat{\lambda} \end{bmatrix} = \begin{bmatrix} c \\ \kappa_0 \end{bmatrix}. \tag{11}$$

The normal matrix in (11) is regular, owing to the relationships of (8). The solution of (11) and its associated dispersion matrix is:

$$\hat{\xi} = \left(N + K^{\mathrm{T}}K\right)^{-1}\left(c + K^{T}\kappa_0\right) = \hat{\xi}_{\mathrm{RLESS}} \tag{12a}$$

$$D\left\{\hat{\xi}\right\} = \sigma_0^2\left(N + K^{\mathrm{T}}K\right)^{-1} N\left(N + K^{\mathrm{T}}K\right)^{-1}. \tag{12b}$$

There is no BLUUE for ξ in the solution space of RLESS; all solutions are biased because of the constraints (datum choice) defined via *K*. However, from RLESS the product $A\hat{\xi}$ does provide the BLUUE of $A\xi$; thus the "corrected" observations are uniformly unbiased and invariant with respect to the chosen datum. Also, the predicted errors and the estimated reference variance are invariant with respect to the chosen datum.[7]

It is natural to seek a minimum bias for the parameters in this solution space of the minimally constrained LESS. The following development of MINOLESS shows a particular minimum constraint that satisfies the minimum bias condition.

2.1.3 MINOLESS and the Equivalent BLUMBE

MINOLESS is the *MInimum NOrm LESS*. It is so called because the estimated parameter vector (i.e., changes to initial coordinate values) has a minimum length amongst all other minimally constrained LESS solutions. In addition to the minimum norm property, it can be shown that MINOLESS yields a minimum trace of the dispersion matrix amongst these LESS solutions. Using a statistical approach, MINOLESS can be derived as the *Best Linear Uniformly Minimum Bias Estimation* (BLUMBE) of ξ (SCHAFFRIN and IZ, 2002). MINOLESS has also been called the "inner constraint" solution by some authors.

To determine MINOLESS, an $l \times m$ matrix *E* having rank *l* is used, where $l = m - q$, and the constraint $E\xi = 0$ is imposed (i.e., some linear combination of the parameters is constrained to zero). *E* is defined such that its transpose forms a basis for the null space (or kernel) of *A*, so that

$$AE^{\mathrm{T}} = 0 \text{ and } \mathcal{R}\left(E^{\mathrm{T}}\right) \overset{\perp}{\oplus} \mathcal{R}\left(A^{\mathrm{T}}\right) = \mathbb{R}^m. \tag{13}$$

The notation of (13) means that not only are the respective column spaces of E^{T} and A^{T} a direct sum of $\mathbb{R}^m$, but also that E^{T} is the orthogonal complement of A^{T} in $\mathbb{R}^m$. The dimensions of the respective column spaces sum to *m* (KOCH, 1999, pg. 13). The Lagrange target function to be minimized is then

$$\Phi\left(\xi,\lambda\right) = e^{\mathrm{T}}Pe + 2\lambda^{\mathrm{T}}\left(E\xi\right) = \underset{(\xi,\lambda)}{\text{stationary}}. \tag{14}$$

The Euler–Lagrange necessary conditions are formed by setting the partial derivatives of (14) equal to zero as follows:

$$\begin{aligned}\frac{1}{2}\frac{\partial \Phi}{\partial \xi} &= N\hat{\xi} - c + E^{\mathrm{T}}\hat{\lambda} \doteq 0 \\ \frac{1}{2}\frac{\partial \Phi}{\partial \lambda} &= E\hat{\xi} \doteq 0.\end{aligned} \tag{15}$$

The sufficient condition is confirmed by $\frac{1}{2}\frac{\partial^2 \Phi}{\partial \xi \partial \xi^{\mathrm{T}}} = N$, which is positive (semi) definite. The equations in (15) can be written in matrix form as

$$\begin{bmatrix} N & E^{\mathrm{T}} \\ E & 0 \end{bmatrix}\begin{bmatrix} \hat{\xi} \\ \hat{\lambda} \end{bmatrix} = \begin{bmatrix} c \\ 0 \end{bmatrix}. \tag{16}$$

The normal matrix in (16) is no longer singular, owing to the complementary sum of $\mathcal{R}\left(E^{\mathrm{T}}\right)$ and $\mathcal{R}\left(A^{\mathrm{T}}\right)$. Considering the properties of E defined above, the solution of (16) reduces to that shown in (17a), and from the law of variance propagation, the dispersion matrix is written in (17b):

$$\begin{aligned}\hat{\xi} &= \left[\left(N + E^{\mathrm{T}}E\right)^{-1} - E^{T}\left(EE^{T}EE^{T}\right)^{-1}E\right]c \\ &= \left(N + E^{\mathrm{T}}E\right)^{-1}c = N^{+}c = \hat{\xi}_{\mathrm{MINOLESS}}\end{aligned} \tag{17a}$$

$$D\left\{\hat{\xi}\right\} = \sigma_0^2\left(N + E^{\mathrm{T}}E\right)^{-1}N\left(N + E^{\mathrm{T}}E\right)^{-1} = \sigma_0^2 N^{+}. \tag{17b}$$

Here, the symbol N^{+} denotes the pseudoinverse (or Moore–Penrose inverse) of N. The pseudoinverse is a special generalized inverse having the following four properties:

$NN^{+}N = N$, $N^{+}NN^{+} = N^{+}$, NN^{+} is symmetric, $N^{+}N$ is symmetric.

It is noted that $\left(N + E^{\mathrm{T}}E\right)^{-1} - E^{\mathrm{T}}\left(EE^{\mathrm{T}}EE^{\mathrm{T}}\right)^{-1}E = N^{+}$; however $\left(N + E^{\mathrm{T}}E\right)^{-1} \neq N^{+}$. The matrix products of (17a) are only equivalent due to multiplication by c. It is also mentioned that, though N^{+} is unique, there are other ways to represent it analytically and other ways to compute it numerically (SCHAFFRIN, 1985, pp. 554,555); however, for a network comprised of GPS baselines only, the formula $\hat{\xi} = \left(N + E^{\mathrm{T}}E\right)^{-1}c$ together with (18) below is quite simple. In this case, the structure of the matrix E is merely

$$E = \left[I_3, \quad \cdots, \quad I_3\right]. \tag{18}$$

It can be shown that the solution in (17a) is equivalent to that derived by beginning with the target function

$$\Phi(\xi,\lambda) = \xi^{\mathrm{T}}\xi + 2\lambda^{\mathrm{T}}(N\xi - c) = \underset{(\xi,\lambda)}{\text{stationary}}, \tag{19}$$

which obviously minimizes the length of ξ, as is required by MINOLESS.[8]

2.1.4 Weighted MINOLESS

In some cases, a priori information about the parameters exists, including stochastic information (variances). Known coordinate variances can be used in the parameter estimation by way of a Weighted MINOLESS solution. Letting P_0 be a positive definite weight matrix for the parameters, and beginning with a target function analogous to (14), where the constraint is now $EP_0\xi = 0$, leads to the following system of normal equations for the Weighted MINOLESS:

$$\begin{bmatrix} N & (EP_0)^{\mathrm{T}} \\ EP_0 & 0 \end{bmatrix} \begin{bmatrix} \hat{\xi} \\ \hat{\lambda} \end{bmatrix} = \begin{bmatrix} c \\ 0 \end{bmatrix}. \tag{20}$$

Here again the matrix on the LHS is nonsingular, since $\mathrm{rk}(EP_0) = \mathrm{rk}(E) = l = \mathrm{rk}((EP_0)E^{\mathrm{T}})$. The solution of (20) and the associated dispersion matrix is

$$\hat{\xi} = \left(N + P_0E^{\mathrm{T}}EP_0\right)^{-1} c = \hat{\xi}_{\text{WMINOLESS}} \tag{21a}$$

$$D\{\hat{\xi}\} = \sigma_0^2 \left(N + P_0E^{\mathrm{T}}EP_0\right)^{-1} N \left(N + P_0E^{\mathrm{T}}EP_0\right)^{-1}. \tag{21b}$$

It can be shown that the solution in (21a) is equivalent to that derived by beginning with the Lagrange target function

$$\Phi(\xi,\lambda) = \xi^{\mathrm{T}}P_0\xi + 2\lambda^{\mathrm{T}}(N\xi - c) = \underset{(\xi,\lambda)}{\text{stationary}}, \tag{22}$$

which obviously minimizes the norm of the weighted parameter vector.[9]

2.1.5 (Weighted) Partial MINOLESS and BLIMPBE

In the work that follows (Chapter 5), it is required to minimize the changes in only a subset of the parameter vector. This can be done using a (Weighted) Partial MINOLESS solution. The Partial MINOLESS model differs from MINOLESS by the use of a selection matrix which picks a subset of the parameter vector for which it is desired to have minimum norm. The solution gives a *best partial trace* of the dispersion matrix among all other minimally constrained LESS solutions. However, the Partial MINOLESS *does not yield a uniformly minimum biased estimate* of the parameters (SCHAFFRIN and IZ, 2002). The minimum (partial) bias characteristic is only realized through the *Best*

LInear Minimum Partial Bias Estimation (BLIMPBE) (SCHAFFRIN and IZ, 2002), which follows Partial MINOLESS below.

1) Partial MINOLESS

Letting lowercase s represent the number of parameters to be selected, and rearranging the order of the parameter vector if necessary, the selection matrix S for the Partial MINOLESS can be written as

$$\underset{m\times m}{S} := \begin{bmatrix} I_s & 0 \\ 0 & 0 \end{bmatrix}, \; s \geq m-q. \tag{23}$$

Of the elements chosen by S, $m-q$ of them must correspond to $m-q$ linearly independent columns of N. In other words, S must successfully remove the network datum deficiency. For the Weighted Partial MINOLESS, the sub–matrix I_s in (23) can be replaced by a weight matrix representing the weights of the selected coordinates. For instance, SP_0S would contain, in lieu of I_s, the respective submatrix of the matrix P_0 introduced in Section 2.1.4, corresponding to the selected parameters, and hence reduced in size to $s\times s$. The constraint criterion is $ES\xi=0$, resp. $E\left(SP_0S\right)\xi=0$. Beginning with a target function analogous to (14), the solution and dispersion for (Weighted) Partial MINOLESS can be expressed as

$$\hat{\xi} = \left(N + SE^{\mathrm{T}}ES\right)^{-1} c = \hat{\xi}_{\mathrm{PMINOLESS}} \tag{24a}$$

$$D\left\{\hat{\xi}\right\} = \sigma_0^2\left(N + SE^{\mathrm{T}}ES\right)^{-1} N\left(N + SE^{\mathrm{T}}ES\right)^{-1}. \tag{24b}$$

2) BLIMPBE

In the development of BLIMPBE by SCHAFFRIN and IZ (2002), a selection matrix $\bar{S}$ is defined as “a suitable positive–semidefinite” matrix, (i.e., $\bar{S}+N$ must be invertible). The solution and dispersion for BLIMPBE given there (ibid.) are

$$\hat{\xi}_{\mathrm{BLIMPBE}} = \left[\bar{S}N\left(N\bar{S}N\bar{S}N\right)^{-} N\bar{S}\right]c \tag{25a}$$

$$D\left\{\hat{\xi}_{\mathrm{BLIMPBE}}\right\} = \sigma_0^2\left[\bar{S}N\left(N\bar{S}N\bar{S}N\right)^{-} N\bar{S}\right]. \tag{25b}$$

The formulae in (25a) and (25b) are invariant with respect to the choice of the g–inverse. SCHAFFRIN and IZ (2002) show that, if the selection matrix is altered so that

$$\bar{S} \rightarrow \left(S+N\right)^{-1}, \tag{26}$$

with S being the same as for the Partial MINOLESS in (23) – (24b), then this “special” BLIMPBE solution yields results identical to that of Partial MINOLESS. Following

SCHAFFRIN and IZ (2002), with a slight modification to the notation, this relationship is expressed as follows:

$$
\begin{aligned}
\hat{\xi}_{\underset{\bar{S}\to(S+N)^{-1}}{\text{BLIMPBE}}} &\to (S+N)^{-1} N \left[N(S+N)^{-1} N (S+N)^{-1} N \right]^{-} N (S+N)^{-1} c \\
&= (S+N)^{-1} N \left[N (S+N)^{-1} N \right]^{-} c \qquad (27) \\
&= \hat{\xi}_{\text{PMINOLESS}}.
\end{aligned}
$$

It can be shown that the second line in (27) fulfills the Partial MINOLESS constraint $ES\xi = 0$, which was used to generate the solution in (24a). Thus we have an intersection of the solution spaces of Partial MINOLESS and BLIMPBE. However, this intersection is subject to the relationship in (26), which is an unnecessary restriction upon the solution space of BLIMPBE. One should ask the more general question: *Is there a minimally constrained LESS which uses a selection (or weight) matrix for the parameters that generates an equivalent solution to BLIMPBE?* So far, it seems that there is not, owing to the loss of uniform minimum bias associated with the minimally constrained LESS solution (with the exception of MINOLESS itself).

It should also be noted that (25a) will not belong to the class of LESS unless $\bar{S}N\left(N\bar{S}N\bar{S}N\right)^{-} N\bar{S} \in \left\{N^{-}\right\}$, which is satisfied if, and only if, $\mathcal{R}\left(N\bar{S}\right) = \mathcal{R}\left(N\right)$. This means that necessarily $\operatorname{rk}\left(N\bar{S}\right) = \operatorname{rk}\left(N\right) \Rightarrow \operatorname{rk}\left(\bar{S}\right) \geq \operatorname{rk}\left(N\right)$ must hold in order for BLIMPBE to belong to the class of LESS. However, this is by no means a sufficient condition, and would not be fulfilled by most $\bar{S}$ selection matrices. Thus, the particular form of $\bar{S}$ may require careful consideration, depending on the objective of the estimation problem at hand. In the work of Chapter 5, the special form of BLIMPBE that generates Partial MINOLESS will be discussed along with a second BLIMPBE solution that uses a different selection matrix entirely.

2.1.6 Adjustment with Stochastic Constraints

The final method of adjustment used in this study incorporates prior information on the parameters by using a priori coordinate variances as stochastic constraints. This is done as an alternative to Weighted MINOLESS and BLIMPBE for comparison purposes. With prior information on all or some of the parameters, the *Adjustment with Stochastic Constraints* (SCLESS) model is written as

$$
\begin{aligned}
\underset{n\times 1}{y} &= \underset{n\times m}{A}\,\xi + e \\
z_0 &= \underset{l\times m}{K}\,\xi + e_0
\end{aligned}, \qquad
\begin{bmatrix} e \\ e_0 \end{bmatrix} \sim \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \sigma_0^2 \begin{bmatrix} P^{-1} & 0 \\ 0 & P_0^{-1} \end{bmatrix} \right), \qquad (28a)
$$

$$
\operatorname{rk}\left(A\right) =: q \leq \left\{m, n\right\},\ \operatorname{rk}(K) =: l \geq m - q,\ \operatorname{rk}\left(\left[A^{\mathrm{T}}, K^{\mathrm{T}}\right]\right) = m. \qquad (28b)
$$

For this study, the positive definite matrix P_0 is the same weight matrix used in the Weighted MINOLESS or BLIMPBE problem, depending on whether all or only a subset of the parameters are weighted. In this model, it is assumed that the reference variance is the same for both e and e_0. The range space of $\left[A^{\mathrm{T}}, K^{\mathrm{T}}\right]$ spans $\mathbb{R}^m$ as is evident from (28b). The redundancy of the system is computed by

$$r := n - m + \operatorname{rk}\left(K\right) = n - m + l\,. \tag{29}$$

It is noted that l and K are defined differently here than in the preceding sections; we now allow $l \geq m - q$. The particular usage of l and K in the adjustments that follow should be apparent from the context.

The Lagrange target function to now minimize is, according to SCHAFFRIN (1995), written as

$$\Phi\left(\xi,\lambda\right) = e^{\mathrm{T}} P e - 2\lambda^{\mathrm{T}}\left(K\xi - z_0\right) - \lambda^{\mathrm{T}} P_0^{-1}\lambda = \underset{(\xi,\lambda)}{\text{stationary}}\,. \tag{30}$$

Upon setting the first derivatives to zero, the Euler–Lagrange necessary conditions are

$$\begin{aligned} \frac{1}{2}\frac{\partial \Phi}{\partial \xi} &= N\hat{\xi} - c + K^{\mathrm{T}}\hat{\lambda} \doteq 0 \\ \frac{1}{2}\frac{\partial \Phi}{\partial \lambda} &= K\hat{\xi} - z_0 - P_0^{-1}\hat{\lambda} \doteq 0, \end{aligned} \tag{31}$$

which, in matrix form, gives the following system of normal equations

$$\begin{bmatrix} N & K^{\mathrm{T}} \\ K & -P_0^{-1} \end{bmatrix}\begin{bmatrix} \hat{\xi} \\ \hat{\lambda} \end{bmatrix} = \begin{bmatrix} c \\ z_0 \end{bmatrix}. \tag{32}$$

The solution and dispersion for the parameter vector with a singular matrix N is

$$\begin{aligned} \hat{\xi} &= \left(N + K^{\mathrm{T}} P_0 K\right)^{-1} c \\ &+ \left(N + K^{\mathrm{T}} P_0 K\right)^{-1} K^{\mathrm{T}}\left[P_0^{-1} + K\left(N + K^{\mathrm{T}} P_0 K\right)^{-1} K^{\mathrm{T}}\right]^{-1}\left(z_0 - K\left(N + K^{\mathrm{T}} P_0 K\right)^{-1} c\right) \end{aligned} \tag{33a}$$

$$D\left\{\hat{\xi}\right\} = \sigma_0^2\left[N + K^{\mathrm{T}} P_0 K\right]^{-1}. \tag{33b}$$

The predicted error vector $\tilde{e}$ is computed as in the first identity of (7a), however its dispersion is different from (7b). This difference is due to the fact that $\left(N + K^{\mathrm{T}} P_0 K\right)^{-1}$ is a generalized inverse for N if and only if $\operatorname{rk}\left(\left[A^{\mathrm{T}}, K^{\mathrm{T}}\right]\right) = \operatorname{rk}(A) + \operatorname{rk}(K)$, which is not required in (28b). The formulae for $\tilde{e}$, $\tilde{e}_0$, and the associated dispersion matrices are as follows:

$$\tilde{e} = y - A\hat{\xi} \tag{34a}$$

$$\tilde{e}_0 = z_0 - K\hat{\xi} \tag{34b}$$

$$\mathrm{D}\{\tilde{e}\} = \sigma_0^2\left(P^{-1} - A\left(N + K^{\mathrm{T}} P_0 K\right)^{-1} A^{\mathrm{T}}\right) = \sigma_0^2 Q_{\tilde{e}} \tag{34c}$$

$$\mathrm{D}\{\tilde{e}_0\} = \sigma_0^2\left(P_0^{-1} - K\left(N + K^{\mathrm{T}} P_0 K\right)^{-1} K^{\mathrm{T}}\right) = \sigma_0^2 Q_{\tilde{e}_0} \tag{34d}$$

$$\mathrm{C}\{\tilde{e}, \tilde{e}_0\} = -\sigma_0^2 A\left(N + K^{\mathrm{T}} P_0 K\right)^{-1} K^{\mathrm{T}} \tag{34e}$$

The model in (28a) and (28b) obviously does not provide RLESS (minimum number of constraints) since it is an over–constrained problem, in general.

2.2 Hypothesis Testing and Outlier Detection

In addition to parameter and dispersion estimations, the models above permit estimation of the reference variance, estimation of observation outliers, and computation of reliability numbers, as well as other quantities of interest. Such quantities are introduced and their formulae given in the following sections. These sections will include the concepts of reliability numbers for correlated observations as well as data snooping and outlier detection at the GPS–baseline–vector level.

2.2.1 Estimated Reference Variance and Global Test of the Adjustment

A value for the reference variance is stated a priori. This value should be known or else assigned based on some legitimate assumption or standard practice. It can also be estimated as a function of the predicted errors, the a priori weight matrix, and the redundancy of the system. Equation (35) gives the formula for the estimated reference variance associated with LESS, which is a *Best Invariant Quadratic Uniformly Unbiased Estimation* (BIQUUE) for σ_0^2 (GRAFAREND and SCHAFFRIN, 1993).

$$\hat{\sigma}_0^2 = \frac{\tilde{e}^{\mathrm{T}} P \tilde{e}}{n - q} \tag{35}$$

Note that for all LESS, $n-q=\mathrm{tr}\left(PQ_{\tilde{e}}\right)$, a relationship that is lost for some cases of BLIMPBE, which is addressed in Section 5.2. For the Adjustment with Stochastic Constraints (Section 2.1.6), the estimated reference variance is written as

$$\hat{\sigma}_0^2=\frac{\tilde{e}^{\mathrm{T}}P\tilde{e}+\tilde{e}_0^{\mathrm{T}}P_0\tilde{e}_0}{n-m+l}. \tag{36}$$

The global test of the adjustment is performed by means of a hypothesis test on the estimated reference variance. This has been called "the most fundamental statistical test in least–squares estimation" by LEICK (1995, pg. 142). The value of the estimated reference variance of (35) is independent of the chosen datum (minimal–constraint). If the observation functional model and the stochastic model are both correct, we would expect $\mathrm{E}\left\{\hat{\sigma}_0^2\right\}=\sigma_0^2$. If the equality is not confirmed by statistical testing, we may suspect that P was chosen incorrectly or the observations contain gross errors or both. The hypothesis test for the global check is

$$\mathrm{H}_0:\mathrm{E}\left\{\hat{\sigma}_0^2\right\}=\sigma_0^2 \text{ versus } \mathrm{H}_\mathrm{a}:\mathrm{E}\left\{\hat{\sigma}_0^2\right\}\neq\sigma_0^2, \tag{37}$$

where σ_0^2 must be specified. H_0 is called the null hypothesis, and H_a is the alternative hypothesis. The test statistic has a chi–square distribution with r degrees of freedom and is written as:

$$T=r\frac{\hat{\sigma}_0^2}{\sigma_0^2}\sim\chi^2\left(r\right), \tag{38}$$

where r is the redundancy of the system as defined above. With a chosen level of significance α, the null hypothesis is accepted if the following inequality holds:

$$\chi^2_{1-\alpha/2}\leq T\leq\chi^2_{\alpha/2}. \tag{39}$$

The far right and left terms are taken from the chi–square tables. If (39) is satisfied, the null hypothesis H_0 is accepted. It is possible that hypothesis testing will lead to the wrong conclusion. If H_0 is rejected when in fact it is true, a Type I error is made. On the other hand, if a false H_0 is accepted, a Type II error is committed. The probability of making a Type I error is α.

2.2.2 Reliability Numbers for Correlated Observations

Each observation in the network contributes a certain amount to the redundancy of the system. This contribution has been called the observation "redundancy number." These numbers have traditionally been used as an aid in identifying potential outliers amongst uncorrelated observations (BAARDA, 1968), hence the alternate name reliability number. The *j*th reliability number r_j is defined as the corresponding diagonal element of the projection matrix $Q_{\tilde{e}}P$, i.e.,

$$r_j = \left(Q_{\tilde{e}}P\right)_{jj}, \text{ with } \operatorname{tr}\left(Q_{\tilde{e}}P\right) = r, \tag{40}$$

which explains the term "redundancy number" for it. Here, $Q_{\tilde{e}}$ is the cofactor matrix associated with the predicted error vector $\tilde{e}$. In the rank deficient GMM, the matrix product $Q_{\tilde{e}}P$ is nothing more than the projection matrix that multiplies y in the computation of $\tilde{e}$, i.e.,

$$\left(Q_{\tilde{e}}P\right) y = \left[I_n - AN^{-}A^{\mathrm{T}}P\right] y = y - A\hat{\xi} = \tilde{e}. \tag{41}$$

The redundancy numbers can be characterized for diagonal *P* by (LEICK, 1995, pg. 162)

$$0 \le r_j \le 1, \quad j \in \{1, \ldots, n\}, \tag{42}$$

a property that is lost in the case of correlated observations. From the inequality of (42), we say that r_j belongs to the unit interval. Redundancy numbers are invariant with respect to the choice of datum. Ideally, each redundancy number would contribute equally to the system redundancy and therefore have a value of $(n-q)/n$. Furthermore, it is said that large values for r_j (i.e., near 1 or at least near the "ideal" value) are an indicator for quality–control potential for uncorrelated observations (SCHAFFRIN, 1997), hence the interpretation as "reliability numbers."

A commonly used estimate for potential outliers is shown in the next section, where the *j*th estimated outlier can be expressed as inversely proportional to the reliability number as defined in (40). In this sense, the reliability number indicates the relative magnitude of the corresponding estimated outlier, with the implication that small reliability numbers make outlier detection difficult. Therefore, analysts typically consider not only the magnitude of the estimated outlier but also the reliability of the observation as reflected in the reliability number, in view of the inequality in (42), when deciding if an observation should be flagged as an outlier. However, since the bounds for r_j shown in (42) only hold in the case of a diagonal weight matrix, this approach may lead to wrong conclusions in the presence of correlated observations, unless the concept of reliability number is redefined, resp. generalized.

For networks that include observed GPS baseline vectors, the weight matrix *P* is not diagonal, and so the reliability number defined in (40) for uncorrelated observations may no longer belong to the unit interval. WANG and CHEN (1994) show that these traditional "redundancy numbers" lead to results that are too optimistic when used with correlated observations. A generalized reliability number (not necessarily belonging to the unit interval) as suggested by WANG and CHEN (1994) has been standardized by SCHAFFRIN (1997) so that the bounding values of (42) are restored. In the following, the jth $n \times 1$ unit vector $\eta_j := [0,\ldots,0,1,0,\ldots,0]^{\mathrm{T}}$ is used in a quadratic form to extract the jth diagonal value from a square matrix. The formula for the generalized reliability number given by WANG and CHEN (1994) is

$$\overline{r}_j = \left(\eta_j^T Q \eta_j\right)\left(\eta_j^T P Q_{\tilde{e}} P \eta_j\right). \tag{43}$$

After the standardization proposed by SCHAFFRIN (1997), the reliability number becomes

$$\overline{\overline{r}}_j = \left(\eta_j^{\mathrm{T}} Q^{-1} \eta_j\right)^{-1}\left(\eta_j^{\mathrm{T}} P Q_{\tilde{e}} P \eta_j\right). \tag{44}$$

The standardized reliability number $\overline{\overline{r}}_j$ belongs to the unit interval. Equations (40), (43), and (44) are equivalent if all observations are uncorrelated. Equation (44) is used for reliability number computations in the analysis in Chapters 4 and 5. It is still open as to how to define reliability numbers in the GMM with Stochastic Constraints (from Section 2.1.6). Perhaps, it is sufficient to implement the cofactor matrix $Q_{\tilde{e}}$ from (34c) into the above formulae, a procedure that is conjectured here.

2.2.3 Studentized Residuals

The stochastic characterization of the GMM given in (1) does not specify a probability density function; only an a priori dispersion matrix for the observations is required to compute the least–squares solution. However, to perform hypothesis testing on the predicted errors, one must specify a probability density function. Experience has shown that errors in surveying observations often tend to be normally distributed. Thus, the assumption may be made that $e \sim \mathcal{N}\left(0, \sigma_0^2 P^{-1}\right)$, which denotes a normal distribution. Analytically, the predicted error in equation (7a) may be rewritten as $\tilde{e} = \left(I_n - AN^{-}A^{\mathrm{T}}P\right)\left(A\xi + e\right)$, which reduces to $\tilde{e} = \left(I_n - AN^{-}A^{\mathrm{T}}P\right)e$. Thus, the predicted error is written as the product of a projection matrix and the true (unknown) vector of errors. Therefore, the assumption of a normal distribution can be extended to the predicted errors (or "residuals"), which is written as $\tilde{e} \sim \mathcal{N}\left(0, \sigma_0^2 Q_{\tilde{e}}\right)$. In practice, the assumption of a normal distribution may be verified by a histogram plot of the predicted errors, resp. scaled residuals.

The term "residual" is introduced here as a synonym to the term "predicted error." Some authors use the term residual to mean "correction" (i.e., opposite sign of error). However, keeping with the sign convention of e in the GMM introduced in (1), the term residual is used here as predicted error. Since the least–squares criterion minimizes the residuals (sum of weighted squares), inspection and evaluation of the residuals is a critical part of the adjustment validation. Depending on the type and relative precision of the observations, the elements of the residual vector may vary significantly in magnitude. Therefore, a means to standardize the residuals is most helpful.

In statistics, a normally distributed sample mean $\bar{x}$, computed from a sample size *n* and having a known value of μ_0 and a standard deviation σ, is transformed to a standardized normal random variable by $z = \left(\bar{x} - \mu_0\right) \Big/ \left(\sigma / \sqrt{n}\right)$ (MIKHAIL and ACKERMANN, 1976, pg. 55). In an analogous manner, the standardized residual for the *j*th observation is written as

$$z_j = \frac{\tilde{e}_j}{\sqrt{\sigma_0^2 \left(Q_{\tilde{e}}\right)_{jj}}}. \tag{45}$$

The double–*j* subscript denotes the *j*th diagonal element of the matrix. Since the reference variance is generally considered to be an unknown quantity, it is replaced by the estimated reference variance (35) or (36) to form the following studentized residual:

$$t_j = \frac{\tilde{e}_j}{\sqrt{\hat{\sigma}_0^2 \left(Q_{\tilde{e}}\right)_{jj}}}. \tag{46}$$

Note that in the case of the GMM with Stochastic Constraints, the residual vector $\tilde{e}_0$ may also be standardized using the diagonal elements of the cofactor matrix $Q_{\tilde{e}_0}$ from (34d). The statistic in (46) is characterized as having a Student's *t* distribution, owing to the random properties of both the numerator and denominator. Studentized residuals are computed and listed in the numerical analysis of Chapters 4 and 5.

2.2.4 Outlier Detection at the GPS–Baseline–Vector Level

Explicit in the GMM is the assumption that the observations contain only random errors without bias. This assumption is expressed as $\mathrm{E}\{e\} = 0$. After the adjustment, we have at our disposal some formulae that we may use to validate our a priori assumptions about the observation errors. For instance, we may assume the presence of one outlier in our data set at a particular observation, estimate this outlier, and then check to see if the estimate is statistically equivalent to zero. If we confirm an outlier value of zero, one observation at a time for every observation, then we may have some assurance that our data set indeed contains only errors of random type without bias (or perhaps gross errors that are too small to detect). BAARDA (1968) presented this procedure as a data snooping technique. Again, it is based on the assumption that only one outlier exists in the data set. This might be somewhat problematic if multiple outliers exist, since the testing of a particular observation with an assumed outlier is no longer tested against an outlier–free data set. However, this procedure is often used in practice and is employed herein as presently described. (Note: ADUOL and SCHAFFRIN (1988) have described a procedure for multiple outlier testing. More recently, GRAFAREND and AWANGE (2002) have proposed a Gauss–Jacobi combinatorial algorithm to detect all outliers in a data set without the presumption of only one outlier being present. This procedure, however, is extremely computer intensive.)

In a geodetic network containing GPS baseline observations, we might like to consider an entire baseline vector as "one" contributing observation. But obviously the observed GPS baseline is comprised of three observational components, which, in a Cartesian parameterization, consist of coordinate differences dX, dY, and dZ. It was already mentioned that Baarda's data snooping algorithm is used to detect outliers in a single observation. This begs the question of what to do with the observed baseline vector if an outlier appears in one or two of the observation components but not in all three. Should the entire observed baseline be flagged for possible rejection or just the components with outliers? There seems to be no basis for using only one or two GPS–baseline observation components while rejecting the other(s), especially when there may be high correlation between the three components (particularly when using a Cartesian parameterization). A *proposed solution* to the problem is to adopt an approach analogous to the single observation testing wherein the entire observed GPS baseline is considered as an individual observation triplet, and thus the test computations are carried out with triples (i.e., vectors) rather than scalars. The two GMM models that lead to the outlier estimate and corresponding test statistic by comparison are as follows:

Model I: Assumed outlier vector in the *k*th observed GPS baseline with this outlier constrained to zero.

$$\underset{n\times 1}{y} = \underset{n\times m}{A}\,\xi + H_k\delta^{(k)} + e,\ e \sim \left(0, \sigma_0^2 P^{-1}\right), \tag{47a}$$

$$0 = \begin{bmatrix} 0 & I_3 \end{bmatrix} \begin{bmatrix} \xi \\ \delta^{(k)} \end{bmatrix} \tag{47b}$$

Here, $\delta^{(k)}$ is a 3 x 1 outlier vector, associated with the *k*th observed GPS baseline, which is immediately set to zero. Let b represent the number of observed GPS baselines vectors in the network, then $k \in \{1,\ldots,b\}$. (In the present case, with a network comprised of only GPS baselines, $b = n/3$.) The matrix H_k is a $3b \times 3$ matrix that, when transposed, can be used to extract the *k*th observed GPS baseline vector from the observation vector y. It is assumed that the observations have been ordered in triples so that each consecutive triple of observations represents a GPS baseline vector. Equation (47b) shows that the outlier has been constrained to zero. This constraint ensures the model will yield estimation results identical to the model given in (1). The following symbol for the P–weighted inner product of $\tilde{e}$ is used later: $\Omega := \tilde{e}^{\mathrm{T}} P \tilde{e}$.

Model II: Assumed outlier in the *k*th observed GPS baseline vector without imposing constraints on its value.

$$\underset{n\times 1}{y} = \underset{n\times m}{A}\,\xi + H_k\delta^{(k)} + e,\ e \sim \left(0, \sigma_0^2 P^{-1}\right) \tag{48}$$

In both Models I and II we still have $\operatorname{rk}(A) =: q \leq \{m, n\}$, and it is noted that $\operatorname{rk}\left(\begin{bmatrix} A, & \mathrm{H}_k \end{bmatrix}\right) = q + 3$. So there is no additional rank deficiency in the system introduced by the additional outlier parameter vector $\delta^{(k)}$ (which, again, has 3 components). For clarity, the form of H_k is shown below.

$$H_k := \begin{bmatrix} 0, & \cdots & 0, & \underset{k\text{th block}}{I_3}, & 0, & \cdots, & 0 \end{bmatrix}^{\mathrm{T}} \tag{49}$$

The least–squares solution of $\delta^{(k)}$ from Model II yields

$$\underset{3\times 1}{\hat{\delta}^{(k)}} = \left[H_k^{\mathrm{T}}\left(PQ_{\tilde{e}}P\right)H_k\right]^{-1} H_k^{\mathrm{T}} P\tilde{e}\,, \tag{50}$$

which represents an estimated outlier triple in the *k*th observed baseline vector. The hypothesis that the expected outlier triple is a vector of zeros is written as

$$\mathrm{H}_0^k : \mathrm{E}\left\{\hat{\delta}^{(k)}\right\} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}} \text{ versus } \mathrm{H}_{\mathrm{a}}^k : \mathrm{E}\left\{\hat{\delta}^{(k)}\right\} \neq \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}. \tag{51}$$

The corresponding test statistic is computed by

$$T_k = \frac{R_k/3}{\left(\mathit{\Omega} - R_k\right)/\left(n - q - 3\right)} \sim F\left(\alpha; 3, n - q - 3\right), \tag{52}$$

with

$$R_k := \hat{\delta}^{(k)^{\mathrm{T}}}\left[H_k^{\mathrm{T}}\left(PQ_{\tilde{e}}P\right)H_k\right]\hat{\delta}^{(k)} \tag{53}$$

and $\mathit{\Omega}$ coming from Model I. The symbol F denotes a Fisher distribution and α the chosen Type I error probability.

The hypothesis test is performed for each of the k observed baseline vectors. The null hypothesis is accepted if

$$T_k \leq F_{(\alpha; 3, r-3)}, \tag{54}$$

where $F_{(\alpha;3,r-3)}$ is the critical value from statistical tables, otherwise the alternative hypothesis is accepted. We would expect to make an error of the first type α percent of the decisions, i.e., reject H_0 when it should have been accepted.

2.2.5 Internal Reliability: Computation of Minimum Detectible Outliers

In addition to estimating outliers and performing hypothesis tests on these estimates, it is important to know what the minimum detectible outlier is for each observed baseline vector. Outliers smaller than the minimum detectible outlier remain in the data set and have an effect on the parameter estimates. Minimum detectible outliers $\delta_{\min}^{(k)}$ may be determined with a certainty of some prescribed value β. When the estimated outlier is less than $\delta_{\min}^{(k)}$, a Type II error is made $1-\beta$ percent of the time, i.e., H_0 of (51) is accepted when it is in fact wrong and should have been rejected; see, e.g., Koch (1999, pg. 280). For a given significance level α and for a given "test power" β, a non–centrality parameter λ' may be determined (e.g., from statistical tables), which can then be used to compute a range for $\delta_{\min}^{(k)}$. The value of λ' also depends upon the degrees of freedom r_1 and r_2, with $r_1 = 3$ being the dimension of the outlier vector and $r_2 = r - 3$, where *r* denotes the redundancy of the system as defined in (2). The applicability to the GMM with Stochastic Constraints remains to be investigated, but is conjectured here via (29).

According to CASPARY (1987, pg. 72), λ' is "the offset of the expectation which the test statistic has to attain, in order that the sample value exceeds the critical value with a probability of $1-\beta$". The formula for the univariate variable is straightforward. However, if the problem of outlier estimation is viewed from the baseline–vector level as described above, investigation of minimum detectible outliers at the vector–triple level is also required. The following is *a proposal for computing the minimum detectible outlier at the baseline–vector level.*

The functional relationship between the minimum detectible outlier and the noncentrality parameter $\left(\lambda' = \lambda'(\alpha, \beta, r_1, r_2)\right)$ is

$$\lambda' = \delta_{\min}^{(k)\mathrm{T}} \left[H_k^{\mathrm{T}} \left(P Q_{\tilde{e}} P \right) H_k \right] \delta_{\min}^{(k)} . \tag{55}$$

The unknown vector $\delta_{\min}^{(k)}$ is of size 3 x 1; thus the problem is underdetermined with one equation and three unknowns. The proposed solution is to apply some subjective, and reasonable, constraint on the vector components. In doing so, typical relative–precisions of GPS–baseline observation components may be considered. From experience, one may consider that the height component is only half as precise as the horizontal components and that the precisions of the horizontal components are equal, i.e., $\sigma_n = \sigma_e = \sigma_{up}/2$.

This relationship has already been seen in the nominal standard error of the CORS coordinates noted in Chapter 1. Translating these relative–precision relationships into outlier vector–component relationships in the local geodetic horizon system (north, east, up), the minimum detectible outlier can be constrained to be

$$\left(\delta_{\min}^{(k)}\right)_{n,e,up} = \gamma\begin{bmatrix}1 & 1 & 2\end{bmatrix}^{\mathrm{T}}, \tag{56}$$

where γ is an unknown scalar to be solved for. Assuming the adjustment has been carried out in the Cartesian system, the vector in (56) must be rotated into the Cartesian system using the following rotational matrix (Rapp, 1993, pg. 152):

$$R = \begin{bmatrix} -\sin\phi\cos\lambda & -\sin\lambda & \cos\phi\cos\lambda \\ -\sin\phi\sin\lambda & \cos\lambda & \cos\phi\sin\lambda \\ \cos\phi & 0 & \sin\phi \end{bmatrix}. \tag{57}$$

Upon rotation, we get

$$\delta_{\min}^{(k)} = R\left(\delta_{\min}^{(k)}\right)_{n,e,up} = \gamma R\begin{bmatrix}1 & 1 & 2\end{bmatrix}^{\mathrm{T}}. \tag{58}$$

The integers in (58) represent relative differences in north, east, and up in the local geodetic horizon coordinate system, with ϕ and λ in (57) being the geodetic coordinates of the "baseline" in said coordinate system. A reasonable choice for ϕ and λ are the mean values of the end points of the baseline vector being considered.

With the constraint of (56) imposed, the vector $\delta_{\min}^{(k)}$ is uniquely determined by solving for the scalar γ

$$\gamma^2 = \frac{\lambda'}{\left(R\begin{bmatrix}1 & 1 & 2\end{bmatrix}^{\mathrm{T}}\right)^{\mathrm{T}}\left[H_k^{\mathrm{T}}\left(PQ_{\tilde{e}}P\right)H_k\right]R\begin{bmatrix}1 & 1 & 2\end{bmatrix}^{\mathrm{T}}} \tag{59}$$

and then substituting into

$$\delta_{\min}^{(k)} = \gamma R\begin{bmatrix}1 & 1 & 2\end{bmatrix}^{\mathrm{T}}. \tag{60}$$

The signs of the components in (56) are arbitrary because the imposed constraints were based on the relative magnitudes of error in north, east, and up; e.g., changing the signs of any component in the vector (56) would result in the same numerical solution for γ in (59).

LEHMER (1944) gives tables for λ' in terms of α, β, r_1, and r_2. The tables list critical values for $\alpha = 0.01, 0.05$ and $\beta = 0.7, 0.8$. In LEHMER's paper, β is defined as "the probability of detecting the falsehood of the hypothesis tested." The tables actually provide values for an auxiliary variable ϕ, and the publication gives a formula for λ' in terms of ϕ and r_1.

2.2.6 External Reliability: Effects of Minimum Detectible Outliers on the Parameter Estimates

External reliability is a measure of the effect of undetected outliers on the estimated parameters. If the parameter solution with an undetected outlier in the *k*th observed baseline is denoted as $\hat{\xi}^{(k)}$, then the difference in the parameter vectors with and without said undetected outlier can be expressed as $\delta = \hat{\xi}^{(k)} - \hat{\xi}$. The normal equations for $\hat{\xi}^{(k)}$ can be expressed as a function of the associated minimum detectible outlier. From the normal equations for $\hat{\xi}^{(k)}$, the *N*–weighted inner product of the difference between $\hat{\xi}^{(k)}$ and $\hat{\xi}$ can be obtained as follows:

$$
\begin{aligned}
&N\hat{\xi}^{(k)} = A^{\mathrm{T}}P\left(y - H_k\delta^k_{(\min)}\right) \\
&\hat{\xi}^{(k)} = N^-_{rs}\left[A^{\mathrm{T}}P\left(y - H_k\delta^k_{(\min)}\right)\right] = \hat{\xi} - N^-_{rs}A^{\mathrm{T}}PH_k\delta^k_{(\min)} \Rightarrow \\
&\hat{\xi}^{(k)} - \hat{\xi} = -N^-_{rs}A^{\mathrm{T}}PH_k\delta^k_{(\min)} \Rightarrow \\
&\left\|\hat{\xi}^{(k)} - \hat{\xi}\right\|^2_N = \left(H_k\delta^k_{(\min)}\right)^{\mathrm{T}}\left(PAN^-_{rs}A^{\mathrm{T}}P\right)H_k\delta^k_{(\min)},
\end{aligned}
$$

where the relation $N^-_{rs}NN^-_{rs} = N^-_{rs}$ given in Section 2.1.1 has been used. It is not difficult to show that the above expression for the weighted inner product is equivalent to

$$
\left\|\hat{\xi}^{(k)} - \hat{\xi}\right\|^2_N = \left(H_k\delta^{(k)}_{\min}\right)^{\mathrm{T}}\left(P - PQ_{\tilde{e}}P\right)H_k\delta^{(k)}_{\min}. \tag{61}
$$

The weighting by *N* is chosen to remove the datum dependency. The square root of (61) is the magnitude of the weighted displacement of the estimate of ξ due to an undetected outlier. It is noted that (61) is unitless.

The quantity in (61) contributes to a change in the quadratic form $\Omega = \tilde{e}^{\mathrm{T}}P\tilde{e}$, and thus also a change in the estimated reference variance. The analytical expression of this change is shown in the following. From 7(a), Ω can also be written as $\Omega = \left(y - A\hat{\xi}\right)^{\mathrm{T}}P\left(y - A\hat{\xi}\right)$, which, after algebraic manipulation, can be expressed as $\Omega = y^{\mathrm{T}}Py - \hat{\xi}^{\mathrm{T}}N\hat{\xi}$. Analogously, the same quadratic form can be written for the solution containing an undetected outlier in the *k*th observed baseline as $\Omega_k = \tilde{e}_k^{\mathrm{T}} P \tilde{e}_k = y^{\mathrm{T}} P y - \left(\hat{\xi} + \delta\right)^{\mathrm{T}} N \left(\hat{\xi} + \delta\right) = y^{\mathrm{T}} P y - \left[\hat{\xi}^{\mathrm{T}} N \hat{\xi} + 2\hat{\xi}^{\mathrm{T}} N \delta + \delta^{\mathrm{T}} N \delta\right]$. The change in the quadratic form, due to the undetected outlier, is then given by the difference $\Delta\Omega = \tilde{e}_k^{\mathrm{T}} P \tilde{e}_k - \tilde{e}^{\mathrm{T}} P \tilde{e} = \delta^{\mathrm{T}} N \delta + 2\hat{\xi}^{\mathrm{T}} N \delta = \left\|\hat{\xi}^k - \hat{\xi}\right\|_N^2 + 2c^{\mathrm{T}} \delta$. When considering the change in the estimated reference variance due to (61), the redundancy of the system and the mixed product $2\hat{\xi}^{\mathrm{T}} N \delta$ must also be taken into account.

2.2.7 Hypothesis Testing of the Estimated Heights

After performing the global test of the estimated reference variance and testing for observation outliers, the parameters may be tested against a priori values, e.g. published coordinates. The entire set of estimated coordinates may be tested at once, or, alternatively, a subset may be tested, including individual testing of the estimated coordinate values. Since heights are of primary interest in this study, they will be tested individually.

Using the symbols $\hat{h}_k$ and h_k^0 for the *k*th estimated and published height values respectively, the hypothesis test for comparing estimated to published values is expressed as

$$\mathrm{H}_0\text{: } \mathrm{E}\left\{\hat{h}_k\right\} = h_k^0 \quad \text{versus} \quad \mathrm{H}_a\text{: } \mathrm{E}\left\{\hat{h}_k\right\} \neq h_k^0 . \tag{62}$$

Note that this is not the same use of *k* as in Sections 2.2.3 and 2.2.4 where it represents the selected baseline number. The test statistic has a Student's *t* distribution and is computed by

$$T_k = \frac{\left|\hat{h}_k - h_k^0\right|}{\sqrt{\hat{\mathrm{D}}\left\{\hat{h}_k\right\}}} \sim t(r) . \tag{63}$$

Here, *r* is used to denote the redundancy of the system as usual, and the symbol $\hat{\mathrm{D}}\left\{\hat{h}_k\right\}$ is the estimated dispersion of the *k*th estimated height, i.e., incorporating $\hat{\sigma}_0^2$ instead of σ_0^2.

For a chosen level of significance α, the null hypothesis is accepted if

$$T_k \le t_{\alpha/2}(r), \tag{64}$$

where the value for the RHS is taken from the statistical tables.

CHAPTER 3

DATA COLLECTION AND PROCESSING

This chapter addresses data collection and processing methods used in the project. A description of the field work and the procedures used for processing the data will be discussed. The software used for computations will also be mentioned.

3.1 Data for CORS Height Validation

The following six CORS were used in the project network: DET1, MIL1, NLIB, SAG1, STB1, and WLCI. These particular CORS were chosen so as to surround the GPS–buoy project region on the east shore of the southern portion of Lake Michigan. In order to introduce a high level of network redundancy, GPS observational data (24–hour sessions) were gathered so that an independent baseline vector connected each CORS to every other CORS in the network (see Figure 1). A set of 15 unique GPS baselines is required to generate the connectivity between the six points (5+4+3+2+1). Since the number of independent baselines for any GPS observation session is one less than the number of observing receivers, only five baselines could be observed from a single observation session using the six CORS. Thus it was necessary to retrieve data from at least three different observation sessions to build up the network. In this experiment, data were taken from five different days to form the network connections.

In an attempt to include data that reflected a range of various satellite constellations and environmental conditions, a total of three complete data sets of 15 observed baselines each were retrieved (i.e., three observed vectors for each baseline depicted in Figure 1). Thus the entire CORS validation network consists of 45 observed baselines comprised of data collected over 15 different days in the year 1999, between day of year (DOY) 64 and DOY 135. The CORS data are available from an NGS web site.[10]

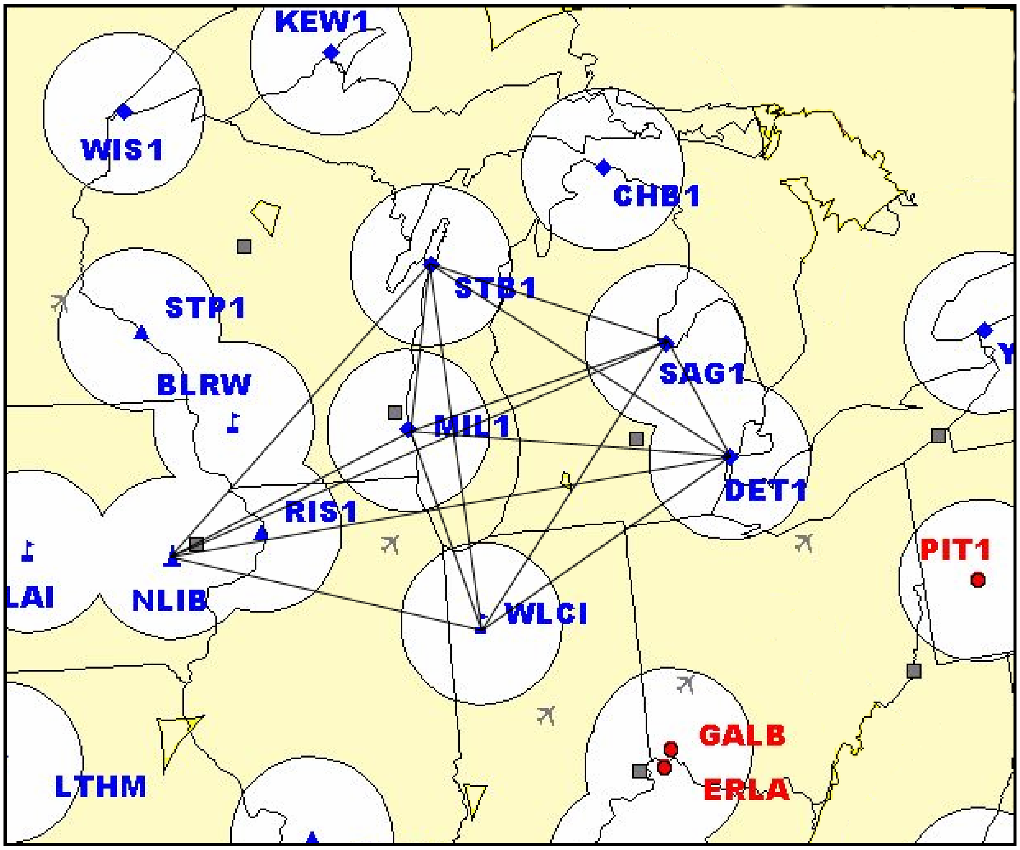


Figure 1: CORS network map

The data DOY associated with each observed baseline is listed in Table 1 (direction of the observed baseline not considered in the table). Published coordinates are given for each station on NGS data sheets as shown in Appendix A.

Baseline	DOY	Baseline	DOY
DET1 – MIL1	65, 80, 133	MLI1 – WLCI	68, 82, 132
DET1 – NLIB	64, 79, 134	NLIB – SAG1	64, 79, 134
DET1 – SAG1	67, 83, 131	NLIB – STB1	64, 79, 134
DET1 – STB1	66, 81, 135	NLIB – WLCI	68, 82, 134
DET1 – WLCI	68, 82, 132	SAG1 – STB1	66, 81, 135
MLI1 – NLIB	64, 79, 134	SAG1 – WLCI	68, 82, 132
MLI1 – SAG1	67, 83, 131	STB1 – WLCI	65, 82, 132
MLI1 – STB1	65, 81, 135		

Table 1: Baseline and data DOY listing

Finally it is noted that the baselines of the CORS height validation network are rather long. Table 2 shows the lengths of baselines in ascending order.

SAG1 → DET1	160	MIL1 → DET1	401
MIL1 → STB1	204	WLCI → SAG1	410
WLCI → MIL1	253	STB1 → DET1	439
STB1 → SAG1	307	WLCI → STB1	443
NLIB → MIL1	333	NLIB → STB1	482
SAG1 → MIL1	336	NLIB → SAG1	666
WLCI → DET1	369	NLIB → DET1	704
WLCI → NLIB	394		

Table 2: Baseline lengths in km

3.2 Field Survey for New Fiducial Points

A field campaign was conducted from June 9, 1999 (DOY 160) to June 11, 1999 (DOY 162) near the eastern shore of Lake Michigan for collection of the data used in the new fiducial point estimation detailed in Chapter 5 (Cheng et al., 2001). The field crew consisted of six participants from OSU and one from NGS. [11] The new fiducial points were actually existing monuments established by the NGS as part of the nationwide spatial reference network; however, the published coordinates are not considered to be as accurate with respect to the ITRF as those of the CORS. Two of the points (BEHD and G317) are constructed of a steel rod driven to a depth of over 20 meters and incased in a protective sleeve with a lid at the surface; the third point is a disk set in a boat–hoist foundation. Information about the points is given in Table 3. A complete description of the points can be retrieved from the NGS database using the PID from Table 3 as a key. Data for the CORS were retrieved from the NGS database via the internet.

Point ID	PID	Rod Depth [m]	Elevation [m], NAVD 88
BEHD	AA8099	21	190.91
G317	OL0372	28	190.565
MBYC	NG0411	disk	177.786

Table 3: New fiducial point data from NGS data base

Following the NGS guidelines for obtaining ±2 cm height accuracy (Zilkowski et al., 1997), and based on advice from NGS personnel, three eight–hour observation sessions were carried out over a three–day period.[12] Observations for two of the three days began at approximately the same time of the day, while the starting time for the third day was offset by four hours. Thus the entire data set spanned a 12–hour segment of a day, thereby permitting the entire GPS constellation, as seen from the occupied stations, to be tracked.

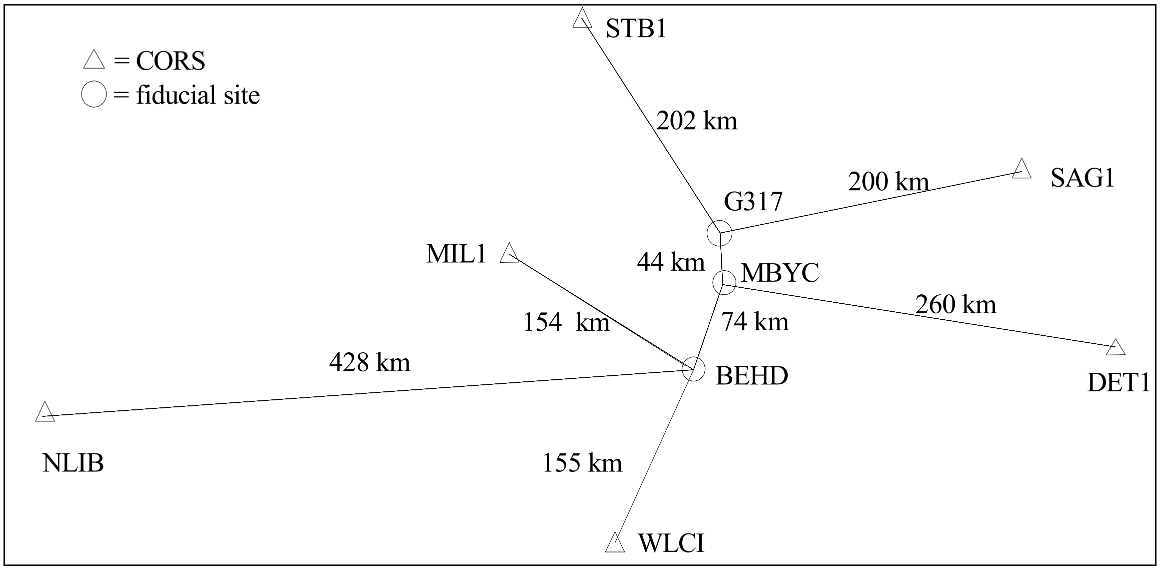


Figure 2: Network diagram for fiducial points

The GPS equipment consisted of Trimble 4000 SSI dual frequency receivers (manufactured by Trimble Navigation Limited of Sunnyvale, CA) and Trimble choke ring antennas. Fixed–height (2 meters) GPS tripods were used to ensure accurate antenna heights, and sand bags were placed at the tripod feet to stabilize the antenna set up. The plumbing apparatus for each tripod was checked for proper adjustment before the work began. The data were downloaded to computers at the end of each observing session for safekeeping. A network diagram showing the connections between the new fiducial points and the CORS, along with approximate baseline distances, is shown in Figure 2.

The figure shows eight baselines. Since data collection was repeated over three days, the total number of observed baselines in the network should have been 24. However, data were not available from station WLCI on DOY 160; so the number of observed baselines in the network for fiducial point determination is 23.

3.3 Data Processing

NGS software, PAGES (Program for Adjustment of GPS Ephemerides), was used to process the GPS data files.[13] Precise GPS orbit ephemeris computed by IGS were used for processing. The PAGES program has the desirable feature of processing all observed baselines in "session mode" so that not only covariances between baseline components are computed, but covariances between all observed baselines in a common session are determined as well. The resulting covariance matrix generated by the PAGES baseline processor becomes the inverse of the weight matrix *P* for the least–squares network adjustment.

Because of session processing, the network weight matrix *P* has many more nonzero elements than the typical diagonal (or 3×3 block diagonal) weight matrix. The diagrams in Figures 3 and 4 provide a visualization of the density of the weight matrices for the observed baselines in the networks of Figures 1 and 2 respectively, by shading in the nonzero elements. The matrix for the CORS validation network has 8 percent (1485/18225) nonzero terms, while the matrix for the second network has 33 percent (216/5184) nonzero elements. This is in contrast to 2.2 percent and 4.2 percent, respectively, for a 3×3 block diagonal matrix used in the case of no correlation between observed baselines. The non–shaded areas in the matrix schematics represent a zero correlation between observation sessions. This implies an absence of correlation in time between successive observation days, which is not actually the case for GPS observations. However, no attempt is made in this work to correlate the sessions with one another. The correlation in time would have less influence on the CORS validation network, as the observations were collected from 15 different days over a span of 72 days (Table 1). Finally, it is noted that given the height of the antenna phase center above the mark, PAGES reduces all observed baseline vectors from the antennas to the marks, which is commonly done in baseline processing algorithms.

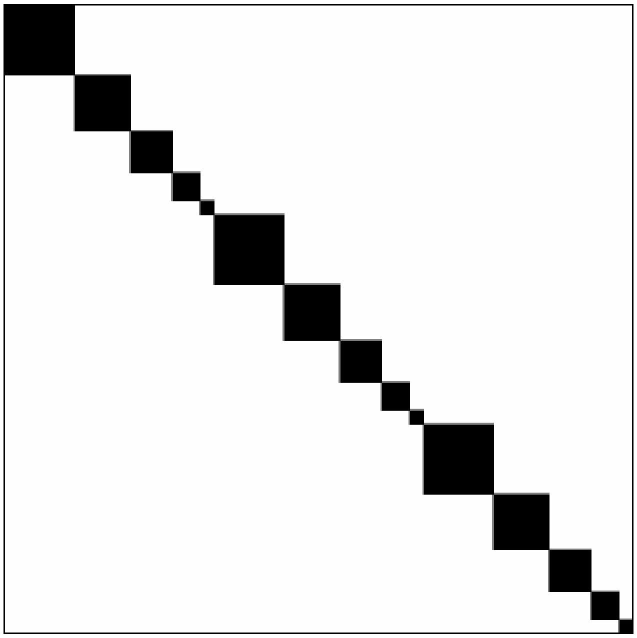


Figure 3: Density of weight matrix for CORS validation network

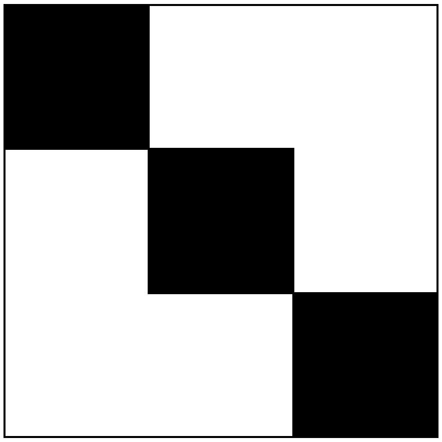


Figure 4: Density of weight matrix for new fiducial point network

All network adjustment computations were performed using routines developed by the author using MATLAB. The MATLAB program will read and parse a priori coordinates, observation records, and weight information. The data files for both networks are listed in Appendices B and C, respectively. Each record begins with a code denoting the type of record. The primary record types for this project are station coordinates, adjustment type, and GPS–baseline observation records. In addition, there are optional records used to indicate a global scale factor for the observation weights and records used to assign centering errors associated with the instrument setups. The following is a brief description of the records that appear in the listings of Appendices B and C.

All fields are space delimited. The symbol $ denotes the beginning of a new data record. The station coordinate record contains fields for the station name, the Cartesian coordinates, and the station standard deviations; the record has the following form:

$$\$\text{XYZ} \quad name \quad X \quad Y \quad Z \quad \sigma_n \quad \sigma_e \quad \sigma_{u.}$$

The standard deviations can be given as any combination of positive real numbers and the characters ! and &, which denote fixed and free respectively. The coordinate system for the coordinate standard deviations is the local geodetic horizon system of the point (north, east, up). The adjustment program propagates these uncertainties into the X,Y,Z coordinate system. Codes for valid adjustment types are listed in Table 4.

Code	Adjustment Type
$RLESS	Restricted LESS
$MINOLESS	Minimum Norm LESS
$WMINOLESS	Weighted Minimum Norm LESS
$PMINOLESS	Partial Minimum Norm LESS
$WPMINOLESS	Weighted Partial Minimum Norm LESS
$BLIMPBE	Best Linear Minimum Partial Bias Estimation
$WBLIMPBE	Weighted Best Linear Minimum Partial Bias Estimation
$SCLESS	Stochastically Constrained LESS
$CLESS	Constrained LESS

Table 4: Valid adjustment–type codes for the network adjustment program

Adjustments requiring a selection matrix must contain the number of points to select as the second and final field of the record (e.g., $PMINOLESS 6). The points specified by this second field are taken from the top of the parameter list; there is no means to select only individual coordinates of a station. The GPS–baseline observation record spans two lines and has the following form:

$$\$\text{GPS} \quad \text{tail} \quad \text{head} \quad dX \quad dY \quad dZ$$
$$\text{var}(dX) \quad \text{covar}(dX,dY) \quad \text{var}(dY) \quad \text{covar}(dX,dZ) \quad \text{covar}(dY,dZ) \quad \text{var}(dZ).$$

Head and tail refer to the ending and beginning baseline station names, respectively. The baseline observation components are given by the coordinate differences dX, dY, dZ. The abbreviations var and covar stand for variance and covariance terms of the baseline observation components. This input format allows for inclusion of data generated by processors that do not return correlations between observed baselines within a common session. A flag in the adjustment program indicates that a complete covariance matrix (based on session processing) is to be read from the computer disk and used instead of the values listed in the data file. A similar option could be employed for the station coordinates in case the weight matrix P_0 was full or at least block diagonal. For this study, P_0 is block diagonal after the transformation of the variances in the local geodetic horizon system to the Cartesian coordinate system. The record $BEGOBS is an indicator to the adjustment program to make intermediate data validation steps before reading the observation data. The # symbol denotes that the line is a comment and should be ignored by the processing algorithms. The record $COVAR_SCALE XX.xx is used to scale the a priori variances/covariances. The following record is used to assign horizontal centering errors and instrument height uncertainties to a station:

$CENTER_ERR name $\sigma_{\text{horizontal}}$ σ_{vertical} .

Name is the station name, and the sigma values refer to horizontal centering standard errors and vertical antenna height (above the mark) standard error, respectively.

CHAPTER 4

CORS HEIGHT VALIDATION

A network comprised only of observed GPS baselines has a datum deficiency of three, owing to the unknown origin parameters of the coordinate system. Thus a datum constraint must be imposed to solve the least–squares normal equations of (5). The resulting coordinate estimates depend directly on the choice of datum. Often the datum is defined by holding three coordinates (X,Y,Z) "fixed." This is the RLESS method discussed in Section 2.1.2. RLESS results in a zero variance for the constrained coordinates and is characterized by error ellipses that grow with distance from the constrained point. Since we wish to test all of the CORS heights, a solution which does not generate a zero variance at any of the points is preferred. As noted in Section 2.1.3 above, MINOLESS generates no zero variances and also yields a minimum–length solution vector and a minimum trace of the dispersion matrix amongst all minimally constrained solutions. Since the MINOLESS solution vector represents the change in coordinate values from the initial approximate values, a solution which is closest to the published coordinates is obtained when the published values are used as the initial approximations (closest in the sense of a minimum norm of the vector of differences between the a priori and the adjusted coordinates).
Table 5 summarizes the published coordinates taken from the data sheets in Appendix A. The abbreviation ARP stands for antenna reference point, and MON stands for monument. Typically the ARP is the bottom surface of the antenna that would mate with, for example, the head of a tripod. For most CORS, the ARP is the primary reference mark that the coordinates are computed for. In the case of station NLIB, the ARP is offset from the monument, as shown in the data sheet.

The published geodetic coordinates refer to ITRF96. As noted in the introduction, NGS does not publish values for the upward component of the CORS velocity vectors. Only station NLIB has a nonzero vertical velocity–component, as computed by the IERS for inclusion in the ITRF (see data sheet in Appendix A). However, the CORS horizontal coordinates should be updated to the project epoch in order not to introduce horizontal displacement biases in the a priori coordinates for the adjustment. A mean (nominal) DOY value of 114 is used for this purpose, corresponding to epoch 1999.312. The published coordinates may then be updated by the formula $\overline{x} = x + dt \cdot v$, where x is the vector of published coordinate values in meters at epoch 1997.0, dt is the difference in epochs in units of years, and v is the published velocity vector in meters per year. The updated coordinates are listed in the last two columns of Table 6, using $dt = 2.312\,\text{yr}$.

The sub–mm deviations in height from the published values listed in Appendix A are attributed to rounding error in the computations. The Cartesian coordinates from the fourth column are used as a priori coordinates in the adjustment (including updates for all three components for station NLIB).

Station	X [m] / Latitude N	Y [m] / Longitude W	Z [m] / Height [m]
DET1 (ARP)	568024.755 42°17′50.45437″	-4690674.635 83°05′43.06542″	4270188.820 145.045
MIL1 (ARP)	172136.032 43°00′09.13101″	-4668696.644 87°53′18.40750″	4327808.348 147.377
NLIB (MON)	-130934.472 41°46′17.72779″	-4762291.729 91°34′29.61729″	4226854.663 207.035
SAG1 (ARP)	496374.994 43°37′43.11958″	-4597431.512 83°50′15.95739″	4378421.351 149.223
STB1 (ARP)	212435.716 44°47′43.74825″	-4528758.901 87°18′51.58610″	4471353.761 148.835
WLCI (ARP)	248645.842 40°48′30.26922″	-4828261.314 87°03′07.14856″	4146460.096 180.424

Table 5: NGS published coordinates ITRF96 (1997.0)

Station Coordinate	$X/Y/Z$ [m] (1997.0)	Velocities [m/yr] $v_X/v_Y/v_Z$	$X/Y/Z$ [m] (1999.312)	ϕ, λ, h (1999.312)
DET1 - X	568024.755	-0.0156	568024.7189	42°17′50.45411″
DEY1 - Y	-4690674.635	-0.0043	-4690674.6449	-83°05′43.06703″
DET1 - Z	4270188.820	-0.0026	4270188.8140	145.0445 m
MIL1 - X	172136.032	-0.0118	172136.0047	43°00′09.13085″
MIL1 - Y	-4668696.644	-0.0019	-4668696.6484	-87°53′18.40870″
MIL1 - Z	4327808.348	-0.0015	4327808.3445	147.3775 m
NLIB - X	-130934.472	-0.0150	-130934.5067	41°46′17.72752″
NLIB - Y	-4762291.729	0.0009	-4762291.7269	-91°34′29.61878″
NLIB - Z	4226854.663	-0.0050	4226854.6514	207.0266 m
SAG1 - X	496374.994	-0.0159	496374.9572	43°37′43.11958″
SAG1 - Y	-4597431.512	-0.0017	-4597431.5159	-83°50′15.95904″
SAG1 - Z	4378421.351	0.0000	4378421.3510	149.2232 m
STB1 - X	212435.716	-0.0164	212435.6781	44°47′43.74796″
STB1 - Y	-4528758.901	-0.0035	-4528758.9091	-87°18′51.58784″
STB1 - Z	4471353.761	-0.0027	4471353.7548	148.8355 m
WLCI - X	248645.842	-0.0149	248645.8076	40°48′30.26911″
WLCI - Y	-4828261.314	-0.0017	-4828261.3179	-87°03′07.15003″
WLCI - Z	4146460.096	-0.0011	4146460.0935	180.4234 m

Table 6: Published ITRF96 (1997.0) and updated coordinates (1999.312)

The matrix Q_0 (inverse of P_0 introduced in Section 2.1.4) contains the a priori variances of the CORS station coordinates. Nominal values are used for five of the CORS, and IERS published values are used for station NLIB. For NLIB the published variances for X, Y, Z are respectively: $(0.002\,\text{m})^2$, $(0.003\,\text{m})^2$, and $(0.003\,\text{m})^2$. However, the velocities used to project NLIB coordinates also have associated variances (see Figure 14 in Appendix A). After propagating the velocity uncertainties into the projected coordinate variances, the a priori variances for NLIB used in the adjustments are: $\sigma_X^2 = (0.00234\,\text{m})^2$, $\sigma_Y^2 = (0.00437\,\text{m})^2$, and $\sigma_Z^2 = (0.00403\,\text{m})^2$. For the other CORS, the nominal values are $\sigma_n^2 = \sigma_e^2 = (0.005\,\text{m})^2$, $\sigma_u^2 = (0.010\,\text{m})^2$. The unknown covariances are set to zero. After propagation of the variances from the n,e,u system into the X,Y,Z system, the Q_0 matrix becomes block diagonal. Table 7 shows the block diagonal entries of Q_0 associated with each station. The data file from Appendix B is used in the CORS Validation adjustment, together with a "session–level" covariance matrix, as described in the following section.

DET1			SAG1		
25.6	-4.9	4.5	25.5	-4.2	4.0
-4.9	65.4	-37.1	-4.2	63.8	-37.2
4.5	-37.1	59.0	4.0	-37.2	60.7
MIL1			STB1		
25.1	-1.5	1.4	25.1	-1.8	1.8
-1.5	65.1	-37.4	-1.8	62.7	-37.5
1.4	-37.4	59.9	1.8	-37.5	62.2
NLIB			WLCI		
5.5	0.0	0.0	25.1	-2.2	1.9
0.0	19.1	0.0	-2.2	67.9	-37.1
0.0	0.0	16.2	1.9	-37.1	57.0

Table 7: Block diagonal elements of Q_0 in units of mm^2 in X,Y,Z system

4.1 CORS Validation Adjustment

As a means to ascertain the quality of the observations and associated a priori weights, the RLESS adjustment is performed first (with station NLIB held fixed). As noted above, RLESS yields BIQUUE for the reference variance; it also yields BLUP for the error vector. It is noted that the constraint matrix K (Section 2.1.2) is weighted by 10^3 in order to maintain numerical stability in the solution. The results of the adjustment are listed in Appendix D. A rather large estimated reference variance value 145.9 was computed. Obviously the alternative hypothesis of (37) is accepted for this estimated reference variance, which warrants further investigation.

It is not uncommon for GPS baseline processing algorithms to return overly optimistic covariance matrices for their estimates. This is likely due in part to the very large formal redundancy in the observation data and the fact that not all systematic errors have been modeled (e.g., atmospheric effects and multipath are difficult to completely model or eliminate), not to mention the often overlooked time–dependent correlation between the observations (and between observation sessions). An overly optimistic covariance matrix Σ returned by the baseline processor, and subsequently used for Q in the network adjustment, will cause the estimated reference variance to be too large. An inspection of the covariance matrix Σ generated by PAGES would seem to indicate overly optimistic values. The following submatrix of Q is associated with the first observed baseline NLIB to MIL1 (see first $GPS record of Appendix B). The values are typical of those for the other observed baselines.

$$ {}^{1,1}Q_{3,3} = \left(10^{-6}\right) \begin{bmatrix} 0.160 & -0.005 & 0.044 \\ -0.005 & 3.240 & -2.713 \\ 0.044 & -2.713 & 2.560 \end{bmatrix} \left[\mathrm{m}^2\right] $$

The largest variance is for dY, which is equivalent to a standard deviation of $10^{-3}\sqrt{(3.24)} = \pm 0.0018\ \mathrm{m}$. Experience would suggest that the standard deviation of GPS baseline observations of the lengths represented in this project are larger than this, possibly by a factor of 10 or more. Furthermore, if the repeated observation values are inspected for this baseline (DOY's 64, 79, 134), differences in the range of –0.025 m to 0.019 m are found, a precision not reflected by Q. Therefore it is reasonable to suspect that the covariance matrix Σ (network adjustment cofactor matrix Q) returned by the baseline processor is too optimistic, and that it should be rescaled. But before doing so, a test for outliers in the observations is required, since the presence of outliers would also inflate the value of the estimated reference variance.

4.2 Outlier Detection and Hypothesis Tests for CORS Adjustments

Outlier estimation and computation of minimum detectible outliers at the GPS–baseline level is performed according to Sections 2.2.4 and 2.2.5. The results are listed in Table 8 below. The estimated outliers are computed according to (50); the test statistic is computed by (52); and equations (59) and (60) are used to compute the minimum detectible outliers. Records for which the null hypothesis is rejected (i.e., equation (54) is not satisfied) are flagged with an asterisk. In keeping with the assumption that only one outlier is present in the data set, the vector having the largest value for the test statistic (number 16) is removed and the adjustment recomputed.

Estimated baseline outliers and minimum detectible outliers in meters.
α = 0.01, β = 0.80, r_1 = 3, r_2 = 117, non-central param. = 8.08,
F(0.01;3,117) = 3.95

Vec#	from	to	est. outlier [dX,dY,dZ]	T_k	min. detect. [dX,dY,dZ]
1	NLIB->	MIL1	[-0.015, 0.010,-0.012]	5.06*	[0.0007,-0.0006, 0.0015]
2	NLIB->	STB1	[0.009, 0.002,-0.000]	1.77	[0.0008,-0.0006, 0.0016]
3	NLIB->	SAG1	[-0.001,-0.024, 0.021]	0.83	[0.0008,-0.0006, 0.0016]
4	NLIB->	DET1	[0.000, 0.016,-0.003]	1.45	[0.0007,-0.0005, 0.0015]
5	WLCI->	STB1	[-0.001, 0.007,-0.008]	0.21	[0.0005,-0.0003, 0.0009]
6	MIL1->	STB1	[-0.001,-0.011, 0.009]	0.64	[0.0004,-0.0003, 0.0008]
7	MIL1->	DET1	[0.006,-0.000, 0.001]	1.08	[0.0005,-0.0004, 0.0010]
8	STB1->	SAG1	[-0.004,-0.003, 0.004]	0.51	[0.0007,-0.0004, 0.0013]
9	STB1->	DET1	[0.016,-0.005, 0.006]	3.93	[0.0007,-0.0004, 0.0014]
10	SAG1->	MIL1	[-0.001, 0.000, 0.001]	0.04	[0.0006,-0.0004, 0.0012]
11	SAG1->	DET1	[0.003,-0.006, 0.005]	0.29	[0.0006,-0.0003, 0.0011]
12	WLCI->	NLIB	[-0.006,-0.018, 0.012]	0.88	[0.0007,-0.0006, 0.0014]
13	WLCI->	MIL1	[0.006,-0.005, 0.004]	2.06	[0.0004,-0.0003, 0.0009]
14	WLCI->	SAG1	[-0.000, 0.006,-0.005]	0.08	[0.0005,-0.0004, 0.0010]
15	WLCI->	DET1	[0.003, 0.001,-0.002]	0.42	[0.0005,-0.0004, 0.0010]
16	NLIB->	MIL1	[0.013,-0.010,-0.004]	6.40*	[0.0007,-0.0005, 0.0014]
17	NLIB->	STB1	[-0.005,-0.008, 0.019]	1.64	[0.0007,-0.0005, 0.0015]
18	NLIB->	SAG1	[0.001, 0.018,-0.017]	0.42	[0.0009,-0.0006, 0.0018]
19	NLIB->	DET1	[-0.018, 0.003, 0.005]	5.22*	[0.0008,-0.0006, 0.0017]
20	DET1->	MIL1	[-0.004,-0.008, 0.013]	0.52	[0.0008,-0.0005, 0.0015]
21	STB1->	MIL1	[-0.001, 0.009,-0.003]	0.42	[0.0006,-0.0004, 0.0012]
22	STB1->	SAG1	[0.004,-0.007, 0.006]	0.62	[0.0005,-0.0003, 0.0010]
23	STB1->	DET1	[-0.001,-0.004,-0.002]	0.57	[0.0006,-0.0004, 0.0011]
24	WLCI->	NLIB	[-0.003, 0.001, 0.004]	0.69	[0.0006,-0.0005, 0.0012]
25	WLCI->	MIL1	[-0.001,-0.002, 0.003]	0.15	[0.0004,-0.0003, 0.0009]
26	WLCI->	STB1	[0.001, 0.002,-0.004]	0.15	[0.0006,-0.0004, 0.0012]
27	WLCI->	SAG1	[0.001, 0.004,-0.002]	0.18	[0.0006,-0.0004, 0.0011]
28	WLCI->	DET1	[0.000,-0.011, 0.003]	1.37	[0.0005,-0.0003, 0.0010]
29	SAG1->	MIL1	[0.001,-0.003, 0.003]	0.03	[0.0006,-0.0004, 0.0013]
30	SAG1->	DET1	[-0.006, 0.001,-0.003]	1.15	[0.0006,-0.0003, 0.0011]
31	SAG1->	MIL1	[0.006,-0.002, 0.004]	0.44	[0.0008,-0.0005, 0.0015]
32	SAG1->	DET1	[-0.005, 0.002,-0.002]	0.74	[0.0005,-0.0003, 0.0011]
33	WLCI->	MIL1	[-0.004, 0.001,-0.002]	0.55	[0.0006,-0.0004, 0.0012]
34	WLCI->	STB1	[0.005, 0.007,-0.004]	0.68	[0.0007,-0.0005, 0.0015]
35	WLCI->	SAG1	[-0.003, 0.004,-0.001]	0.23	[0.0007,-0.0005, 0.0014]
36	WLCI->	DET1	[-0.004,-0.006, 0.012]	1.31	[0.0007,-0.0005, 0.0014]
37	DET1->	MIL1	[0.003, 0.002, 0.003]	0.25	[0.0009,-0.0006, 0.0017]
38	NLIB->	MIL1	[-0.002, 0.007,-0.006]	0.31	[0.0005,-0.0004, 0.0010]
39	NLIB->	STB1	[0.002,-0.008, 0.006]	0.16	[0.0007,-0.0005, 0.0014]
40	NLIB->	WLCI	[0.000,-0.004,-0.000]	0.16	[0.0006,-0.0005, 0.0013]
41	NLIB->	SAG1	[-0.002,-0.013, 0.008]	0.60	[0.0006,-0.0004, 0.0012]
42	NLIB->	DET1	[0.006, 0.010,-0.004]	1.47	[0.0006,-0.0005, 0.0012]
43	STB1->	MIL1	[-0.000,-0.005,-0.004]	0.99	[0.0005,-0.0003, 0.0010]
44	STB1->	SAG1	[-0.006,-0.001, 0.000]	1.08	[0.0006,-0.0004, 0.0011]
45	STB1->	DET1	[0.000, 0.003,-0.002]	0.03	[0.0005,-0.0003, 0.0011]

Table 8: CORS estimated outliers, test statistics, and minimum detectible outliers

After removal of vector 16, the adjustment yields the following two vectors for which the null hypothesis is rejected.

```
1   NLIB->MIL1 [-0.014, 0.009,-0.013]  5.08* [ 0.0007,-0.0006, 0.0015]
9   STB1->DET1 [ 0.016,-0.005, 0.006]  4.54* [ 0.0007,-0.0004, 0.0014]
```

Vector 1 is flagged again. Since it has the larger test statistic value, it is removed and vector 16 is included again for a new computation. The flagged vectors (numbered to retain their original numbers from Table 8) follow.

```
9   STB1->DET1 [ 0.016,-0.005, 0.006] 4.39* [ 0.0007,-0.0004, 0.0014]
16  NLIB->MIL1 [ 0.012,-0.010,-0.005] 6.40* [ 0.0007,-0.0005, 0.0014]
19  NLIB->DET1 [-0.018, 0.004, 0.004] 5.66* [ 0.0008,-0.0006, 0.0017]
```

The following vectors are flagged when both vectors 1 and 16 are removed from the adjustment.

```
9   STB1->DET1 [ 0.016,-0.005, 0.006] 5.11* [ 0.0007,-0.0004, 0.0014]
19  NLIB->DET1 [-0.015, 0.003, 0.002] 4.32* [ 0.0008,-0.0006, 0.0017]
```

Vector 9 is then removed along with 1 and 16 and the adjustment recomputed. Only one vector is flagged.

```
19  NLIB->DET1 [-0.014, 0.003, 0.002] 4.39* [ 0.0008,-0.0006, 0.0017]
```

Since vector 19 was flagged in the initial adjustment, it is removed by itself for yet another adjustment computation.

```
1   NLIB->MIL1 [-0.015, 0.010,-0.012] 5.50* [ 0.0007,-0.0006, 0.0015]
9   STB1->DET1 [ 0.015,-0.005, 0.006] 3.99* [ 0.0007,-0.0004, 0.0014]
16  NLIB->MIL1 [ 0.011,-0.010,-0.003] 4.98* [ 0.0007,-0.0005, 0.0014]
```

Finally it is concluded that vectors 1, 9, 16, and 19 may be considered as outliers and removed from the data set. It is also noted that the reliability numbers listed in the last column of Appendix D show that the smallest value associated with the components of the four suspect vectors is 0.92. This indicates that the measurements were well controlled and further supports removing the vectors from the data set. It is noted that vectors 1, 16, and 19 each originated at NLIB, with 1 and 16 representing the same baseline, and vectors 16 and 19 are from the same day (DOY 79). NLIB is the only station in the network distant from the Great Lakes environment, which may account for different atmospheric influences that were not modeled in the baseline processing. Again, the long length of the baselines as shown in Table 2 is mentioned.

The method of outlier estimation and detection at the baseline–vector level may be compared to the baseline–component method by use of the studentized residuals discussed in Section 2.2.3. Using a component–wise approach, the hypothesis test of (51) is modified so the components of the estimated outlier $\hat{\delta}^{(k)}$ of equation (50) are tested one at a time using the studentized residual t_j of equation (46) as the test statistic (for *k*th observed baseline and *j*th observed baseline component, respectively). For a given significance level α, the null hypothesis is rejected if the magnitude of the studentized residual exceeds the critical value of the Student's *t* distribution, i.e., H_0^j is rejected if $\left|t_j\right| > t_{(\alpha/2,\,r)}$. At the 0.01 significance level, the critical value for the complete data set is $t_{(\alpha=0.01/2,\,r=120)} = 2.617$. All records for which the critical value is exceeded are flagged with an asterisk in the adjustment results of Appendix D. It is interesting to note that vectors 1, 16, and 19 were flagged in the initial adjustment using the baseline–vector method (Table 8), and components from vectors 1, 9, 13, 19 were flagged using the baseline–component method (Appendix D). Furthermore, with four vectors removed (1, 9, 16, and 19), the baseline–vector method produced no further flagged records; while the baseline–component method flagged a component of vector 11 (see Appendix E). Thus, by experiment it has been shown that testing at the baseline level rather than the component level can produce different conclusions in outlier testing.

With the four baseline outliers removed, the adjustments yields 96.2 for the value of the estimated reference variance. This corresponds to overly optimistic uncertainties by a factor of about 10 at the standard deviation level, which seems apparent from inspection of the PAGES covariance matrix and considering the repeat values of the observed baselines. Therefore, the adjustment cofactor matrix Q is scaled a priori by a constant factor of 96. This is not to say that the a priori reference variance is changed; it must remain set to unity in order that the assumption of a common reference variance for P and P_0 in the model with stochastic constraints (28a) remains valid. After the rescaling of Q, the adjustment yields an estimated reference variance of 1.0. This leads to the acceptance of the null hypothesis of (37), with the inequality (39) expressed numerically as $\chi^2_{1-\alpha/2} = 81.1 \le T = 108.2 \le \chi^2_{\alpha/2} = 138.7$ (documented on the first page of Appendix E).

Owing to the invariant properties of the adjusted observations and the estimated reference variance already mentioned, the results of hypothesis testing conducted for RLESS with station NLIB fixed holds for (Weighted) MINOLESS as well. The steps of outlier detection were not repeated in the Adjustment with Stochastic Constraints. However, outliers and minimum detectible outliers were computed in that adjustment using the data set of 41 observed baseline vectors. The values are similar to those computed for Weighted MINOLESS (see Appendix F). Histogram plots of the predicted errors and the studentized residuals for the minimally constrained adjustments are shown in Figures 5 and 6, respectively. The graphs are superimposed with a fitted normal–density curve.

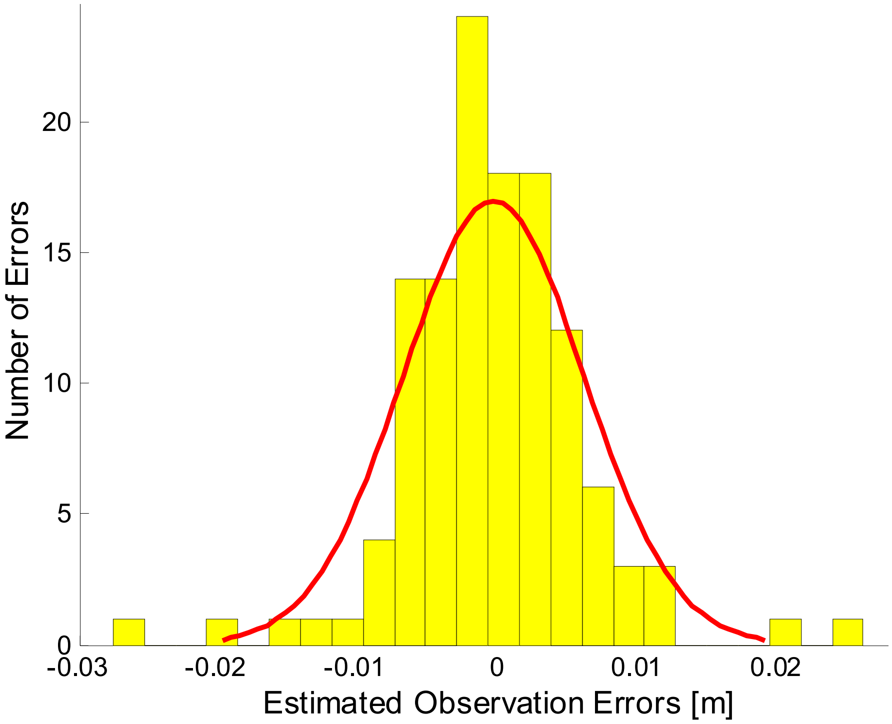


Figure 5: Predicted–error histogram for RLESS CORS adjustment, 41 observed baseline vectors

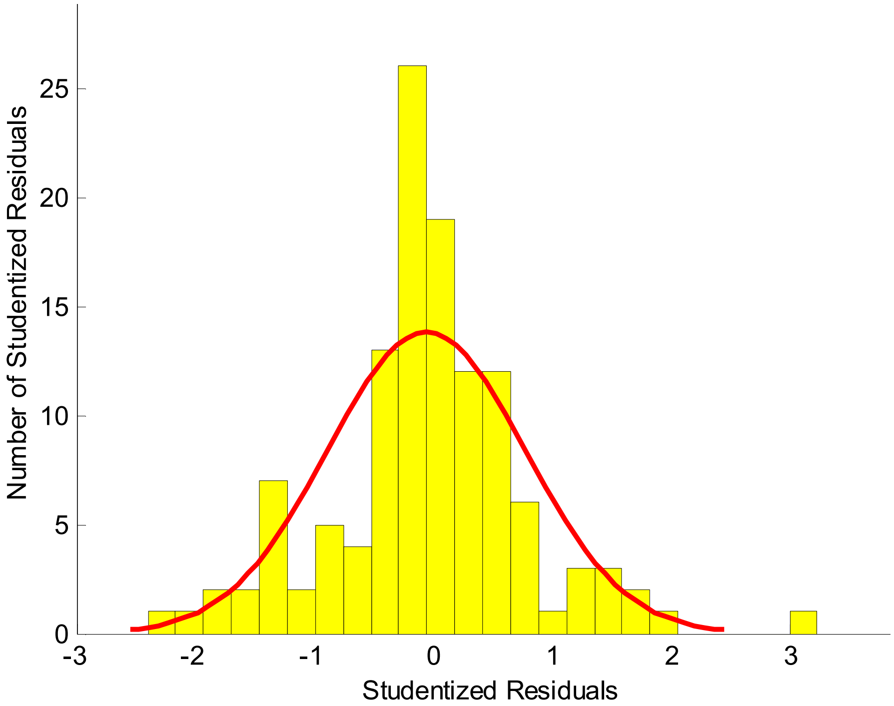


Figure 6: Studentized–residual histogram for RLESS CORS adjustment, 41 observed baseline vectors

The histogram plots show a more–or–less normal distribution of the errors, which lends credence to the assumption of a normal distribution made for hypothesis testing. Both the traditional “redundancy” numbers (40) and the standardized reliability numbers (44) were computed and listed in the last two columns, respectively, of Appendix D. The difference in magnitude between the two quantities only varies by about five percent. However, the differences could be much greater for a data set with stronger correlation between the observations. Thus, it is recommended that the standardized reliability numbers be adopted for correlated observations.

After removal of the four suspect vectors, the external reliability was computed for each of the 41 observed baseline vectors and listed in Table 9. The square root of the tabulated values represents the magnitude of the displacement of ξ (weighted by N) due to the presence of an undetected outlier in the corresponding observed GPS vector. The largest value is 3.127. These values are unitless and should be considered in a relative sense, especially compared to the quadratic form $\Omega = \tilde{e}^{\mathrm{T}} P \tilde{e}$, which is 108.2 for this adjustment.

Undetected Outlier in Vector#	from		to	External Network Reliability
1	NLIB	->	STB1	0.643
2	NLIB	->	SAG1	0.513
3	NLIB	->	DET1	0.627
4	WLCI	->	STB1	2.893
5	MIL1	->	STB1	1.852
6	MIL1	->	DET1	1.172
7	STB1	->	SAG1	0.985
8	SAG1	->	MIL1	0.917
9	SAG1	->	DET1	0.883
10	WLCI	->	NLIB	2.032
11	WLCI	->	MIL1	1.730
12	WLCI	->	SAG1	1.373
13	WLCI	->	DET1	1.150
14	NLIB	->	STB1	0.836
15	NLIB	->	SAG1	0.485
16	DET1	->	MIL1	0.613
17	STB1	->	MIL1	0.776
18	STB1	->	SAG1	1.288
19	STB1	->	DET1	0.845
20	WLCI	->	NLIB	3.127
21	WLCI	->	MIL1	1.706
22	WLCI	->	STB1	1.079
23	WLCI	->	SAG1	0.987
24	WLCI	->	DET1	1.194
25	SAG1	->	MIL1	0.719
26	SAG1	->	DET1	0.912
27	SAG1	->	MIL1	0.529
28	SAG1	->	DET1	1.027
29	WLCI	->	MIL1	0.702
30	WLCI	->	STB1	0.605
31	WLCI	->	SAG1	0.520
32	WLCI	->	DET1	0.488
33	DET1	->	MIL1	0.460
34	NLIB	->	MIL1	1.206
35	NLIB	->	STB1	0.764
36	NLIB	->	WLCI	1.248
37	NLIB	->	SAG1	0.848
38	NLIB	->	DET1	0.697
39	STB1	->	MIL1	1.617
40	STB1	->	SAG1	0.872
41	STB1	->	DET1	0.851

Table 9: CORS external reliability values from RLESS

4.3 Comparison of RLESS, MINOLESS, Weighted MINOLESS, and Adjustment with Stochastic Constraints

After reducing the original data set from 45 to 41 observed baseline vectors and rescaling the a priori cofactor matrix by 96, station coordinates were estimated using RLESS, MINOLESS, Weighted MINOLESS, and Adjustment with Stochastic Constraints

(SCLESS). Coordinates for station NLIB were held fixed in the RLESS solution. The results are tabulated in the tables below. Table 10 lists the estimated geodetic coordinates for each solution type. It is interesting to note that WMINOLESS and SCLESS yielded the same values for the estimate coordinates within the precision of the survey, which is not necessarily expected. Table 11 lists the changes from the a priori coordinates (1999.312 epoch) rotated into the local geodetic horizon system, where it can be seen that the MINOLESS solution yielded the smallest overall change as compared to the other minimum constraint solutions (RLESS and WMINOLESS). Table 12 shows the estimated standard deviations in north, east, and up. These values are the positive square roots of the diagonal elements of the estimated dispersion matrix rotated into the north, east, up system, i.e., $\hat{\sigma}_j = \sqrt{\left(\hat{D}\left\{\hat{\xi}_{n,e,u}\right\}\right)_{jj}}$. The difference between the dispersion and the estimated dispersion matrices (shown with a hat over the *D*) is that the latter uses the estimated reference variance as opposed to the a priori value. Table 12 also lists the estimated reference variance, the trace of the estimated dispersion matrix, and the RMS of the respective coordinates. Hypothesis testing of the estimated heights is addressed in the next section.

	RLESS	MINOLESS	WMINOLESS	SCLESS
DET1	42°17′50.45429″ -83°05′43.06695″ 145.0446 m	42°17′50.45418″ -83°05′43.06689″ 145.0461 m	42°17′50.45419″ -83°05′43.06691″ 145.0456 m	42°17′50.45419″ -83°05′43.06691″ 145.0459 m
MIL1	43°00′09.13089″ -87°53′18.40903″ 147.3683 m	43°00′09.13079″ -87°53′18.40896″ 147.3696 m	43°00′09.13080″ -87°53′18.40899″ 147.3691 m	43°00′09.13080″ -87°53′18.40898″ 147.3706 m
NLIB	41°46′17.72753″ -91°34′29.61879″ 207.0266 m	41°46′17.72743″ -91°34′29.61871″ 207.0280 m	41°46′17.72743″ -91°34′29.61874″ 207.0274 m	41°46′17.72744″ -91°34′29.61874″ 207.0271 m
SAG1	43°37′43.11955″ -83°50′15.95894″ 149.2208 m	43°37′43.11944″ -83°50′15.95887″ 149.2223 m	43°37′43.11945″ -83°50′15.95890″ 149.2217 m	43°37′43.11945″ -83°50′15.95890″ 149.2222 m
STB1	44°47′43.74804″ -87°18′51.58779″ 148.8377 m	44°47′43.74793″ -87°18′51.58771″ 148.8390 m	44°47′43.74794″ -87°18′51.58774″ 148.8385 m	44°47′43.74794″ -87°18′51.58774″ 148.8382 m
WLCI	40°48′30.26949″ -87°03′07.15037″ 180.4242 m	40°48′30.26939″ -87°03′07.15030″ 180.4257 m	40°48′30.26939″ -87°03′07.15033″ 180.4252 m	40°48′30.26938″ -87°03′07.15032″ 180.4252 m

Table 10: CORS estimated geodetic coordinates (ϕ, λ, h)

		RLESS	MINOLESS	WMINOLESS	SCLESS
DET1	*dn*	-5.5	-2.2	-2.4	-2.4
	de	-2.1	-3.5	-2.9	-2.8
	du	-0.1	-1.6	-1.1	-1.4
MIL1	*dn*	-1.1	2.1	1.9	1.9
	de	7.5	5.8	6.5	6.3
	du	9.3	7.9	8.4	6.9
NLIB	*dn*	0.0	3.2	3.0	2.8
	de	0.0	-1.9	-1.2	-1.0
	du	0.0	-1.3	-0.8	-0.5
SAG1	*dn*	1.1	4.4	4.2	4.1
	de	-2.5	-4.0	-3.4	-3.4
	du	2.4	1.0	1.6	1.1
STB1	*dn*	-2.1	1.2	1.0	0.9
	de	-1.3	-2.9	-2.3	-2.3
	du	-2.2	-3.5	-2.9	-2.6
WLCI	*dn*	-1.7	-8.5	-8.7	-8.3
	de	7.9	6.3	7.0	6.7
	du	-0.8	-2.4	-1.8	-1.8
norm		20.1	17.6	17.8	16.6
mean	*dn*	-1.6	0.0	-0.2	-0.2
	de	1.6	0.0	0.6	0.6
	du	1.4	0.0	0.6	0.3

Table 11: CORS changes from a priori coordinates (*dn*, *de*, *du*) in units of mm

	RLESS $\hat{\sigma}_0^2 = 1.00$			MINOLESS $\hat{\sigma}_0^2 = 1.00$			WMINOLESS $\hat{\sigma}_0^2 = 1.00$			SCLESS $\hat{\sigma}_0^2 = 0.96$		
	$\mathrm{tr}\left(\hat{D}\{\hat{\xi}\}\right) = 396 \cdot 10^{-6}\,\mathrm{m}^2$			$\mathrm{tr}\left(\hat{D}\{\hat{\xi}\}\right) = 140 \cdot 10^{-6}\,\mathrm{m}^2$			$\mathrm{tr}\left(\hat{D}\{\hat{\xi}\}\right) = 186 \cdot 10^{-6}\,\mathrm{m}^2$			$\mathrm{tr}\left(\hat{D}\{\hat{\xi}\}\right) = 200 \cdot 10^{-6}\,\mathrm{m}^2$		
Station	$\hat{\sigma}_n$	$\hat{\sigma}_e$	$\hat{\sigma}_u$	$\hat{\sigma}_n$	$\hat{\sigma}_e$	$\hat{\sigma}_u$	$\hat{\sigma}_n$	$\hat{\sigma}_e$	$\hat{\sigma}_u$	$\hat{\sigma}_n$	$\hat{\sigma}_e$	$\hat{\sigma}_u$
DET1	1.4	2.0	8.5	0.7	0.8	3.9	0.7	1.2	5.5	2.1	1.9	5.1
MIL1	1.4	1.5	8.4	0.7	0.6	3.8	0.7	0.8	5.4	2.0	1.8	5.1
NLIB	0.1	0.0	0.0	1.0	1.2	6.3	1.0	0.8	3.6	2.1	1.7	3.7
SAG1	1.4	1.9	8.4	0.7	0.7	3.9	0.7	1.1	5.4	2.1	1.8	5.1
STB1	1.6	1.6	8.7	0.9	0.7	4.5	0.9	0.9	5.9	2.1	1.8	5.4
WLCI	1.5	1.6	9.1	1.0	0.7	5.1	1.0	0.9	6.4	2.2	1.8	5.8
RMS	**1.3**	**1.6**	**7.9**	**0.8**	**0.8**	**4.7**	**0.8**	**1.0**	**5.4**	**2.1**	**1.8**	**5.1**

Table 12: CORS estimated standard deviations (*n*, *e*, *u*) in units of mm

4.4 Hypothesis Testing for CORS Heights

Hypothesis testing of the estimated heights is carried out in accordance with Section 2.2.7, with the goal being to test if the estimated heights, based on observations from the project epoch (1997.312), agree with published height values referring to the 1997.0 epoch. The hypothesis testing is done for each of the four solution types. It is noted again that the height of NLIB has been projected forward via the IERS published velocity vectors; all other heights in the vector h^0 of (62) refer to the 1997.0 epoch. For a redundancy of 108 and a significance level of 0.05, the critical value of the Student's t distribution is $t_{(\alpha=0.05/2, r=108)} = 1.982$. For the stochastically constrained solution, the system redundancy is 123 and the critical value is 1.979. Test–statistic values are listed in Table 13 for each solution type. Only the value for station MIL1 in the MINOLESS adjustment exceeded the Student's *t*–distribution critical value, though MIL1 values are also larger than usual for the other solution types. However, a reduction in magnitude of only 0.5 mm between the estimated and the a priori value for MIL1 would have decreased the test statistic to less than the critical value. With the exception of this one case, the null hypothesis of (62) is accepted for each station in all four adjustment methods.

Station	RLESS	MINOLESS	WMINOLESS	SCLESS
DET1	0.017	0.418	0.193	0.278
MIL1	1.097	**2.052**	1.556	1.416
NLIB	0.000	0.213	0.232	0.138
SAG1	0.287	0.253	0.289	0.218
STB1	0.253	0.767	0.499	0.501
WLCI	0.093	0.459	0.280	0.327

Table 13: Test–statistic values for CORS height hypothesis test

4.5 Summary of CORS Adjustments

Four of the original 45 observed baseline vectors were flagged as outliers and removed from the final data set. Given the considerable length of the baselines and the possibility that station NLIB may have been susceptible to environmental influences different from those of the other stations near the Great Lakes, this does not seem to be an unusually large number of rejections. The numerical results show that MINOLESS yields a smaller length of parameter vector (smallest overall change from a priori coordinate values) and a smaller trace of the dispersion matrix as compared to RLESS, which was expected. Also, from the last part of Table 11, it is seen that the coordinate changes using MINOLESS were zero in an average sense. However, for MINOLESS the null hypothesis of (62) was "narrowly rejected" for station MIL1. RLESS is the least desirable of the four solutions, since only five of the six points absorb the larger dispersion–matrix trace in their variances. Both the Weighted MINOLESS and the Adjustment with Stochastic Constraints are appealing in that they incorporate a priori variance information about the parameters. The author would argue that, in general, the Weighted MINOLESS is preferred, not only because it handles a priori covariance information about the parameters but also because of its minimum constraint characteristic.

The final conclusion is that the published height values from the 1997.0 epoch (with NLIB transformed to 1999.312 via the velocity vector) agree substantially with observations made at the 1999.312 epoch. Therefore, these published values will be used in the estimation of the new fiducial points addressed in Chapter 5. It should be noted that the testing of the published coordinates with respect to later GPS baseline observations can really only validate that the *height differences* are statistically unchanged. Any constant vertical shift over the whole network region, for example long–wave post glacial rebound phenomena, could not be detected by this method. An undetected constant change in height over the entire region could be significant for scientific studies. However, the testing conducted herein is valuable in that it indicates there have not been local vertical deformations that have significantly changed any of the station heights with respect to the others. Oftentimes, vertical deformations are strongly dependent upon local phenomena (e.g., aquifer compression).

For the record, results of the final adjustments of the 41 observed baseline vectors are listed in Appendices E and F for the Weighted MINOLESS and SCLESS, respectively.

CHAPTER 5

COORDINATE ESTIMATION OF NEW (FIDUCIAL) POINTS

The second part of the project treats the estimation of the coordinates (ellipsoidal heights in particular) of the new GPS–buoy fiducial sites. The station names for the new points are BEHD, G317, and MBYC. Coordinates for the new fiducial sites will be estimated by the method of RLESS, BLIMPBE, and by Adjustment with Stochastic Constraints (SCLESS), with a comparison between the results of each. In this network, only observed vectors associated with the baselines depicted in Figure 2 are used; data from the CORS validation adjustment are not considered. Furthermore, the original published horizontal coordinate values (1997.0 epoch) are now projected forward to the 1999.442 epoch, which corresponds to the nominal mean observation DOY 161.5 (including updates for all three components for station NLIB). Table 14 shows the coordinates of the CORS at the published and project epochs. The sub–mm deviations in height from the published values listed in Appendix A are attributed to rounding error in the computations. The Cartesian coordinates from the fourth column are used as a priori coordinates in the adjustment.

Station - Coordinate	$X/Y/Z$ [m] (1997.0)	Velocities [m/yr] $v_X/v_Y/v_Z$	$X/Y/Z$ [m] (1999.442)	ϕ, λ, h (1999.442)
DET1 - X	568024.755	-0.0156	568024.7169	42°17′50.45410″
DEY1 - Y	-4690674.635	-0.0043	-4690674.6455	-83°05′43.06713″
DET1 - Z	4270188.820	-0.0026	4270188.8137	145.0446 m
MIL1 - X	172136.032	-0.0118	172136.0032	43°00′09.13085″
MIL1 – Y	-4668696.644	-0.0019	-4668696.6486	-87°53′18.40877″
MIL1 – Z	4327808.348	-0.0015	4327808.3443	147.3775 m
NLIB - X	-130934.472	-0.0150	-130934.5086	41°46′17.72752″
NLIB - Y	-4762291.729	0.0009	-4762291.7268	-91°34′29.61887″
NLIB - Z	4226854.663	-0.0050	4226854.6508	207.0262 m
SAG1 - X	496374.994	-0.0159	496374.9552	43°37′43.11958″
SAG1 - Y	-4597431.512	-0.0017	-4597431.5162	-83°50′15.95914″
SAG1 - Z	4378421.351	0.0000	4378421.3510	149.2233 m
STB1 - X	212435.716	-0.0164	212435.6760	44°47′43.74795″
SYB1 - Y	-4528758.901	-0.0035	-4528758.9095	-87°18′51.58794″
STB1 - Z	4471353.761	-0.0027	4471353.7544	148.8355 m
WLCI - X	248645.842	-0.0149	248645.8056	40°48′30.26910″
WLCI - Y	-4828261.314	-0.0017	-4828261.3182	-87°03′07.15012″
WLCI - Z	4146460.096	-0.0011	4146460.0933	180.4234 m

Table 14: Published ITRF96 (1997.0) and updated coordinates (1999.442)

Since observations to the new fiducial points required the use of tripods to center the GPS antennas over the marks, the introduction of centering errors into the observational stochastic model at these stations is appropriate. Based on experience with the particular type of tripod used and on the accuracy of centering apparatus, a centering error of $\pm 0.003\,\mathrm{m}$ with respect to the horizontal axes is adopted. Since the "fixed–height" tripods are manufactured with a precise 2–meter dimension from the tip of the centering staff to the antenna ARP surface, the height of GPS antenna above the mark is considered an errorless quantity in the adjustment. Representing the horizontal centering variances along the north and east local–horizon axes as $\sigma_n^2 = \sigma_e^2 = (0.003\,\mathrm{m})^2$ and using the rotational matrix of (57), the centering errors (assumed uncorrelated) in the horizontal plane are transformed into the X,Y,Z parameter coordinate–system by variance propagation. The resulting (full) 3×3 *cofactor* matrix is added to the corresponding block–diagonal sub–matrix of Q. The 3×3 matrix is referred to as a cofactor matrix here in order to imply an associated reference variance identical with that used for the observed GPS vectors. The addition of the 3×3 matrix is made once for each observed vector that either originates or terminates at a station with centering errors, and the addition is made twice if both ends of the vector are at stations having centering errors.

5.1 Estimation of Fiducial Point Heights Using RLESS

The results of the minimum constraint adjustment RLESS (12a) are used to evaluate the quality of the observations. Appendix G contains a listing of the RLESS adjustment results using the 23–vector data set. The adjustment yields 12.86 for the estimated reference variance (35) and flags baseline vectors 9 and 17 as potential outliers according to Section 2.2.4. Table 15 shows a listing of the estimated and minimum detectible outliers computed in accordance with equations (50) and (60), respectively. The table also shows the test statistic computed by (52). Vectors number 9 and 17 are marked with an asterisk since the computed test statistic exceeds the critical value of the *F*–distribution, and (54) is not satisfied. After removing vector 17, the larger outlier, the estimated reference variance reduces to 8.82, and no further vectors are flagged as outliers. Still the null hypothesis for the test of the estimated reference variance (37) is rejected. A rescaling of the observation cofactor matrix *Q*, similar to that discussed in Section 4.2, is again necessary to consider.

```
Estimated baseline outliers and minimum detectible outliers in meters.
α = 0.01, β = 0.80, r1= 3, r2 = 42, non-central parameter = 8.90,
F(0.01;3,42) = 4.285
Vec# from  to   est. outlier[dX,dY,dZ]   Tk     min. detect. [dX, dY, dZ]
 1  MBYC->G317 [ 0.001,-0.001,-0.012] 0.32  [ 0.0037,-0.0025, 0.0073]
 2  SAG1->G317 [-0.002, 0.005, 0.013] 0.38  [ 0.0047,-0.0030, 0.0093]
 3  DET1->MBYC [ 0.019,-0.000, 0.002] 0.62  [ 0.0050,-0.0033, 0.0098]
 4  BEHD->MBYC [-0.014, 0.012, 0.003] 0.45  [ 0.0040,-0.0028, 0.0080]
 5  NLIB->BEHD [-0.003,-0.024,-0.000] 0.72  [ 0.0055,-0.0043, 0.0112]
 6  MIL1->BEHD [-0.013,-0.001, 0.007] 0.38  [ 0.0045,-0.0032, 0.0090]
 7  G317->STB1 [ 0.000, 0.005,-0.020] 0.81  [ 0.0045,-0.0030, 0.0091]
 8  NLIB->BEHD [-0.007,-0.004, 0.003] 0.07  [ 0.0059,-0.0047, 0.0121]
 9  MIL1->BEHD [-0.027, 0.036,-0.011] 4.67* [ 0.0048,-0.0034, 0.0096]
10  MBYC->BEHD [-0.000,-0.000, 0.010] 0.17  [ 0.0043,-0.0030, 0.0086]
11  G317->MBYC [-0.003, 0.004,-0.001] 0.05  [ 0.0041,-0.0028, 0.0081]
12  SAG1->G317 [-0.004, 0.000,-0.004] 0.05  [ 0.0051,-0.0033, 0.0101]
13  DET1->MBYC [-0.005,-0.021, 0.022] 1.38  [ 0.0055,-0.0036, 0.0107]
14  STB1->G317 [ 0.001, 0.007,-0.012] 0.30  [ 0.0049,-0.0032, 0.0098]
15  BEHD->WLCI [-0.009, 0.024,-0.009] 0.62  [ 0.0065,-0.0048, 0.0129]
16  NLIB->BEHD [ 0.011, 0.032,-0.005] 1.14  [ 0.0058,-0.0046, 0.0120]
17  MIL1->BEHD [ 0.041,-0.040, 0.005] 7.87* [ 0.0050,-0.0036, 0.0100]
18  MBYC->BEHD [-0.014, 0.013,-0.007] 0.87  [ 0.0043,-0.0030, 0.0086]
19  G317->MBYC [ 0.004,-0.003,-0.012] 0.24  [ 0.0040,-0.0027, 0.0080]
20  SAG1->G317 [ 0.005,-0.004,-0.010] 0.27  [ 0.0052,-0.0033, 0.0104]
21  DET1->MBYC [-0.016, 0.025,-0.026] 2.05  [ 0.0056,-0.0036, 0.0109]
22  STB1->G317 [-0.001, 0.001,-0.011] 0.20  [ 0.0048,-0.0032, 0.0097]
23  BEHD->WLCI [ 0.009,-0.024, 0.009] 0.62  [ 0.0065,-0.0048, 0.0129]
```

Table 15: Estimated outliers, test statistics, and minimum detectible outliers

Because of the increase in Q from the centering errors, the matrix Q cannot simply be scaled by the estimated reference variance of the initial adjustment. And because the scaling is based upon the assumption that the covariance matrix associated with the observed GPS baselines (i.e., as determined by PAGES) is overly optimistic, the scaling must take place before Q is increased by the cofactors from the centering errors. It is logical to assume that the scale factor should be of the same order of magnitude as that determined in Section 4.2 for the CORS Validation adjustment. Perhaps it should be about one–third to one–half the magnitude, owing to the shorter observations sessions (8 hours instead of 24) and the lower network redundancy (42 compared to 108, with outlier vectors removed). The following excerpt of the first 3×3 block–diagonal portion of Q (before centering errors are considered) gives a representative example of the a priori observational cofactors:

$${}^{1,1}Q_{3,3} = \left(10^{-6}\right)\begin{bmatrix} 0.160 & -0.325 & 0.290 \\ -0.325 & 5.290 & -4.575 \\ 0.290 & -4.575 & 4.410 \end{bmatrix}\left[\mathrm{m}^2\right].$$

The square roots of the diagonal elements represent the precisions of the observed baseline–vector components as determined by the baseline processor. The average square–root value is 1.6 mm, which is arguably too small by one order of magnitude. While the choice of the particular scale–factor value to use is somewhat subjective, a value of 48 was chosen, which is one half the value used in the CORS Validation adjustment (Section 4.2).

The subsequent RLESS adjustment yields 1.02 for the estimated reference variance, which leads to an acceptance of the null hypothesis (37). Histogram plots of the predicted errors and studentized residuals for the “outlier–free” adjustment are shown in Figures 7 and 8, respectively. The graphs show that the errors are somewhat peaked in the center with a couple of high bars at the edges; overall the deviation from the superimposed normal curve is not too radical.

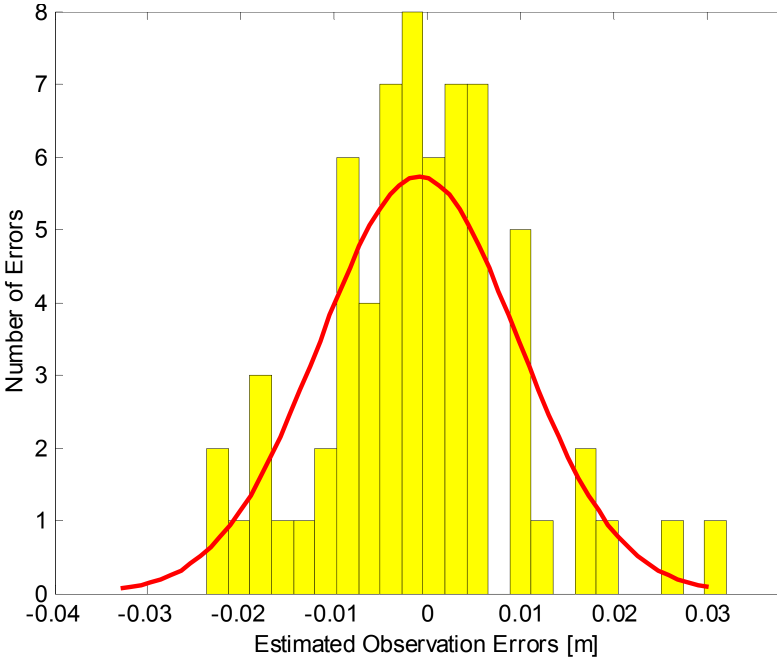


Figure 7: Predicted–error histogram for RLESS adjustment, 22 observed baseline vectors

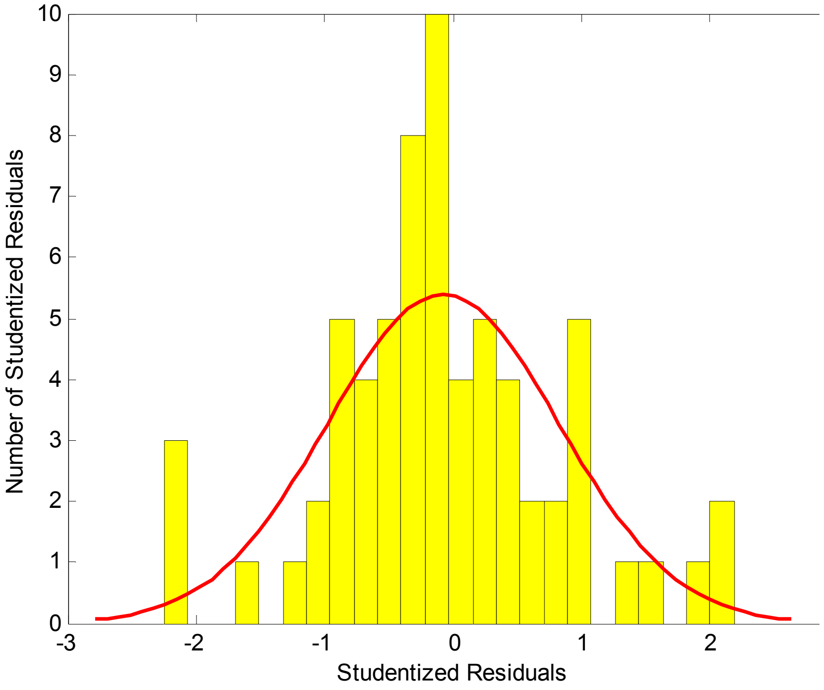


Figure 8: Studentized–residual histogram for RLESS adjustment, 22 observed baseline vectors

5.2 Estimation of Fiducial Point Heights Using BLIMPBE and Adjustment with Stochastic Constraints

This section includes the selecting/weighting of a subset of points in the process of estimating the coordinates of the new points. Two BLIMPBE solutions using different types of selection matrices and the Adjustment with Stochastic Constraints (SCLESS) are computed and compared.

In order to identify the two BLIMPBE solutions, the one based on the first type of selection matrix will hereinafter be referred to as WBLIMPBE, for "Weighted" BLIMPBE. And the solution based on the second type of selection matrix will retain the original label BLIMPBE. (This is done only as a convenience, and is not meant to imply the introduction of a new estimator.) For WBLIMPBE, the $\bar{S}$ matrix of (26) is used (i.e., $\bar{S} \rightarrow (S+N)^{-1}$), with the six CORS stations selected. In addition, the submatrix I_s of (23), as used in (26), is replaced by P_0; thus the stochastic information about the control points are incorporated as well (hence the choice of "Weighted" in the label). As discussed in Section 2.1.5, the numerical solution based on this form of $\bar{S}$ is equivalent to that of Partial MINOLESS.

Now, for the BLIMPBE solution, a "standard" selection matrix as defined in (23) is used for $\bar{S}$. Note that this choice for $\bar{S}$ will result in a zero value in $\hat{\xi}$ for all elements in the $s+1$ through m locations, as is evident by inspection of (25a) (based on the previous assumption that the parameter vector has been arranged so that the selected points appear first). In other words, the a priori coordinate values for the non–selected points will be retained (the so–called "reproducing" property). Likewise, from (25b), it can be seen that the variances of the non–selected points are zero. Consequently, for this BLIMPBE solution it is now the new points that are selected! This is in contrast to typical use of the selection matrix where the control points are selected (e.g., Partial MINOLESS). *The results of using such a selection matrix can be interpreted as having a minimum bias for the new points, rather than for the control points.* Finally, it is noted that the denominator of (35) must be altered to account for the change in the value of $\operatorname{tr}(Q_{\tilde{e}}P)$, which is no longer the system observational–redundancy $n-q$. This modification should also be reflected in the hypothesis test (52) for the estimated outlier. The value used for the denominator is determined by starting with the definition $\mathrm{E}\{\tilde{e}^T P\tilde{e}\} := tr(Q_{\tilde{e}}P)\sigma_0^2$ and proceeding as follows.

$$\begin{aligned}
\mathrm{E}\{\tilde{e}^T P\tilde{e}\} &:= tr(Q_{\tilde{e}}P)\sigma_0^2 \\
&= \sigma_0^2 \operatorname{tr}\left(I_n - A\bar{S}N\left(N\bar{S}N\bar{S}N\right)^{-} N\bar{S}A^{\mathrm{T}}P\right) \\
&= \sigma_0^2 \operatorname{tr}\left(I_n - \left(N\bar{S}N\bar{S}N\right)^{-} N\bar{S}\left(A^{\mathrm{T}}PA\right)\bar{S}N\right) && \text{trace invariant to cyclic transformation} \\
&= \sigma_0^2 \left(n - \operatorname{rk}\left(\left(N\bar{S}N\bar{S}N\right)^{-} N\bar{S}N\bar{S}N\right)\right) && \text{trace of idempotent matrix is rank of matrix} \\
&= \sigma_0^2 \left(n - \operatorname{rk}\left(N\bar{S}N\bar{S}N\right)\right) && \text{because } \operatorname{rk}(A^{-}A) = \operatorname{rk}(A)\text{, see KOCH (1999, pg. 51)}
\end{aligned}$$

$$= \sigma_0^2 \left(n - \text{rk}\left(N\bar{S} \right) \right)$$

$$\Rightarrow \hat{\sigma}_0^2 = \tilde{e}^T P \tilde{e} \Big/ \left(n - \text{rk}\left(N\bar{S} \right) \right), \text{ where typically } \text{rk}\left(N\bar{S} \right) \le \text{rk}\left(A \right) = q \,.$$

The SCLESS adjustment uses the same P_0 matrix used in WBLIMPBE. The weights are generated from the values shown in Table 7. The same a priori scaling of the cofactor matrix as described in Section 5.1 is done for all solutions in this section.

Some comparisons of the characteristics of the residuals are shown in Table 16. The BLIMPBE solution generates the largest range of residuals and comes closer to the SCLESS values then to the WBLIMPBE. The variation in the distribution of residuals can be seen from the histogram plots in Figures 9, 10, and 11.

	BLIMPBE	WBLIMPBE	SCLESS
minimum	-3.42	-3.01	-3.19
maximum	6.52	3.74	6.06
range	9.94	6.75	9.25
rms	1.61	1.15	1.47

Table 16: Residual statistics in units of cm

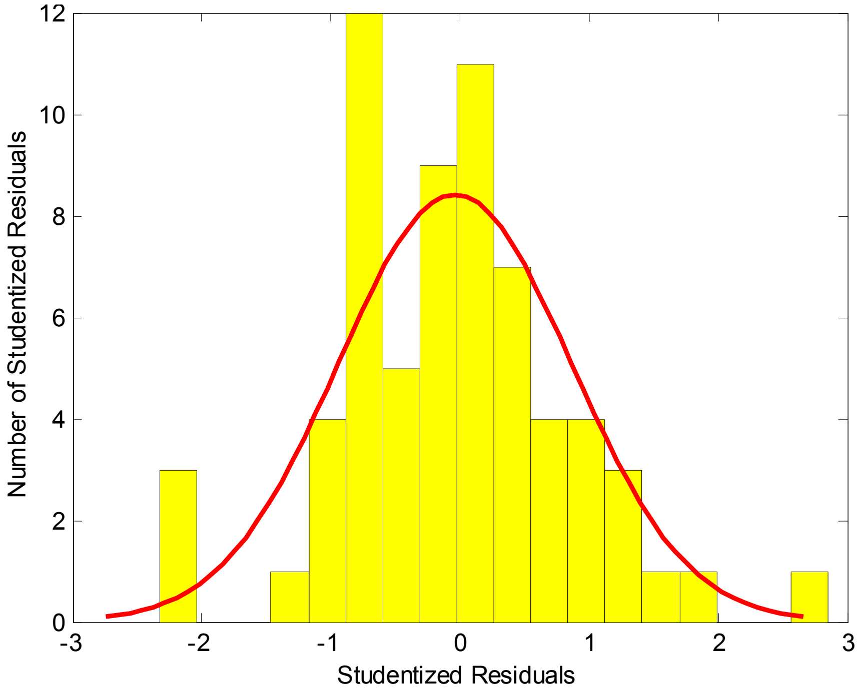


Figure 9: Studentized–residual histogram for BLIMPBE adjustment, 22 observed baseline vectors

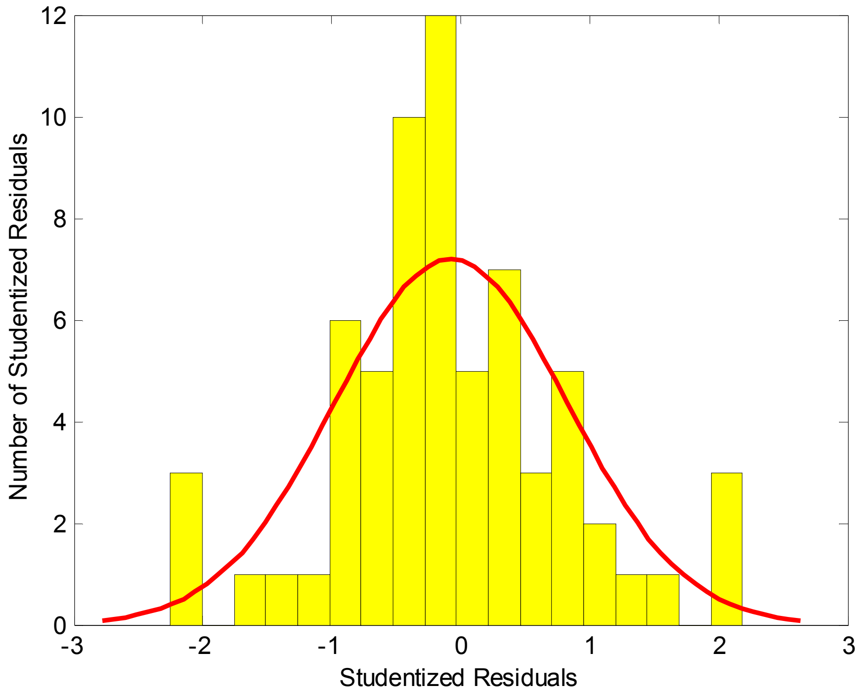


Figure 10: Studentized–residual histogram for WBLIMPBE adjustment, 22 observed baseline vectors

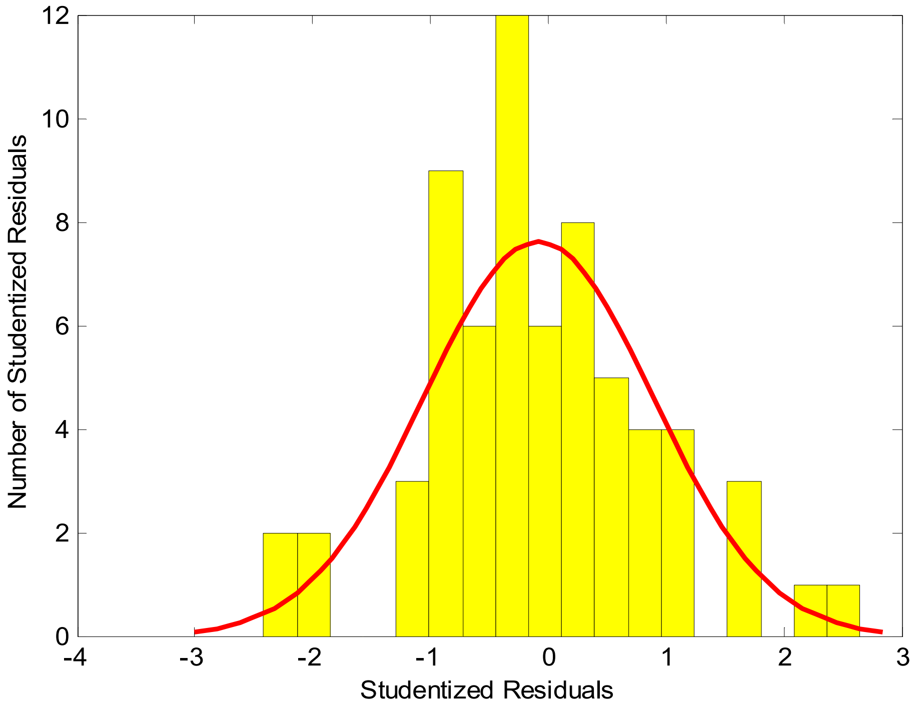


Figure 11: Studentized–residual histogram for SCLESS adjustment, 22 observed baseline vectors

In addition to estimated outliers at the baseline–vector level, minimum detectible outliers (50) were also computed for BLIMPBE, WBLIMPBE, and SCLESS. The values are tabulated in the respective appendices. Differences between minimum detectible outliers computed in each solution are given in Table 17. The difference is in the sense of SCLESS solution minus W/BLIMPBE solutions. Overall, the WBLIMPBE yields results closer to that of SCLESS than does BLIMPBE.

The external reliability numbers for the W/BLIMPBE and SCLESS solutions are also listed in the respective appendices. The WBLIMPBE solution yields the smaller value for each vector. As discussed in Section 2.2.6, the quadratic form $\Omega = \tilde{e}^{\mathrm{T}} P \tilde{e}$ is directly affected by the presence of an undetected outlier, which is reflected in the value of the external reliability number for the corresponding observation. Therefore, the external reliability values should be considered together with the appropriate denominator of (35), (36), or the value computed for BLIMPBE, when evaluating the impact on the estimated reference variance. These denominator values are 54, 42 and 57 for the BLIMPBE, WBLIMPBE and SCLESS solutions, respectively. The respective values for Ω are 72.769, 42.708 and 56.251.

		SCLESS – BLIMPBE				SCLESS – WBLIMPBE			
Vector No.	Baseline	ΔdX [mm]	ΔdY [mm]	ΔdZ [mm]	norm [mm]	ΔdX [mm]	ΔdY [mm]	ΔdZ [mm]	norm [mm]
1	MBYC→G317	0.9	-0.6	1.7	1.8	-1.0	0.8	-2.1	2.2
2	SAG1→G317	1.6	-0.9	3.1	3.4	-0.9	0.7	-1.9	2.0
3	DET1→MBYC	1.5	-1.1	3.0	3.4	-1.2	0.7	-2.3	2.4
4	BEHD→MBYC	0.8	-0.6	1.6	1.7	-1.4	1.0	-2.9	3.2
5	NLIB→BEHD	1.5	-1.2	3.1	3.6	-3.5	2.7	-7.2	11.9
6	MIL1→BEHD	2.0	-1.4	4.0	4.9	-1.6	1.1	-3.2	3.7
7	G317→STB1	1.4	-0.8	2.7	2.9	-0.6	0.5	-1.3	1.3
8	NLIB→BEHD	1.2	-1.0	2.6	2.9	-1.0	0.8	-2.0	2.2
9	MIL1→BEHD	1.7	-1.2	3.4	4.0	-1.1	0.7	-2.1	2.2
10	MBYC→BEHD	0.8	-0.5	1.5	1.6	-0.7	0.5	-1.5	1.5
11	G317→MBYC	0.8	-0.6	1.7	1.8	-0.7	0.5	-1.4	1.4
12	SAG1→G317	1.4	-0.9	2.8	3.1	-0.6	0.4	-1.2	1.2
13	DET1→MBYC	1.1	-0.8	2.2	2.4	-0.6	0.4	-1.1	1.1
14	STB1→G317	1.2	-0.8	2.4	2.6	-0.5	0.3	-0.9	0.9
15	BEHD→WLCI	2.4	-1.8	4.7	6.4	-2.3	1.7	-4.5	6.0
16	NLIB→BEHD	1.3	-1.0	2.6	2.9	-1.2	1.0	-2.4	2.7
17	MBYC→BEHD	0.7	-0.5	1.3	1.3	-0.7	0.5	-1.5	1.5
18	G317→MBYC	0.8	-0.6	1.7	1.8	-0.7	0.4	-1.3	1.3
19	SAG1→G317	1.1	-0.8	2.3	2.5	-0.5	0.3	-1.0	1.0
20	DET1→MBYC	1.1	-0.7	2.2	2.3	-0.7	0.4	-1.3	1.3
21	STB1→G317	0.9	-0.6	1.9	2.0	-0.5	0.3	-0.9	0.9
22	BEHD→WLCI	2.1	-1.6	4.1	5.3	-2.5	1.8	-4.8	6.6
	min	0.9	-0.6	1.7	1.8	-3.5	0.3	-7.2	0.9
	max	1.6	-0.9	3.1	3.4	-0.5	2.7	-0.9	11.9
	range	1.5	-1.1	3.0	3.4	3.0	2.4	6.3	11.0
	avg	0.8	-0.6	1.6	1.7	-1.1	0.8	-2.2	2.7

Table 17: Difference in minimum detectible outliers (SCLESS – W/BLIMPBE)

Table 18 shows the estimated geodetic coordinates for each solution. As an aid to viewing the differences between the solutions, Table 19 gives the CORS station coordinates from the four solutions as expressed in the local geodetic horizon system of each of the respective CORS (a priori coordinates), thereby showing the changes from the CORS a priori coordinate values. The norm values in the last column of each solution type in Table 19 represent the change in each point from the a priori coordinates, whereas the norm values on the bottom row show the changes of all the CORS coordinates along the respective axes. The bold values are the total norm of the coordinate changes. This table shows the reproducing property of BLIMPBE when using the "standard" selection matrix of (23). Table 20 lists the W/BLIMPBE coordinates for the new points as expressed in the local geodetic horizon system of SCLESS station coordinates, which highlights the differences in coordinate estimates between the W/BLIMPBE solutions and those of SCLESS. Note that both BLIMPBE and WBLIMPBE closely match the horizontal coordinates of SCLESS, but the heights of BLIMPBE are much closer to SCLESS than are those of WBLIMPBE.

	RLESS	BLIMPBE	WBLIMPBE	SCLESS
DET1	42°17′50.45439″ -83°05′43.06656″ 145.0073 m	42°17′50.45410″ -83°05′43.06713″ 145.0446 m	42°17′50.45414″ -83°05′43.06676″ 145.0252 m	42°17′50.45414″ -83°05′43.06696″ 145.0416 m
MIL1	43°00′09.13116″ -87°53′18.40883″ 147.3249 m	43°00′09.13085″ -87°53′18.40877″ 147.3775 m	43°00′09.13090″ -87°53′18.40896″ 147.3430 m	43°00′09.13089″ -87°53′18.40893″ 147.3687 m
NLIB	41°46′17.72752″ -91°34′29.61887″ 207.0262 m	41°46′17.72752″ -91°34′29.61887″ 207.0262 m	41°46′17.72726″ -91°34′29.61895″ 207.0445 m	41°46′17.72739″ -91°34′29.61887″ 207.0281 m
SAG1	43°37′43.11977″ -83°50′15.95891″ 149.2142 m	43°37′43.11958″ -83°50′15.95914″ 149.2233 m	43°37′43.11950″ -83°50′15.95911″ 149.2319 m	43°37′43.11950″ -83°50′15.95925″ 149.2326 m
STB1	44°47′43.74840″ -87°18′51.58766″ 148.8055 m	44°47′43.74795″ -87°18′51.58794″ 148.8355 m	44°47′43.74813″ -87°18′51.58781″ 148.8232 m	44°47′43.74804″ -87°18′51.58787″ 148.8348 m
WLCI	40°48′30.26942″ -87°03′07.14981″ 180.3673 m	40°48′30.26910″ -87°03′07.15012″ 180.4234 m	40°48′30.26919″ -87°03′07.14995″ 180.3856 m	40°48′30.26918″ -87°03′07.15003″ 180.4183 m
BEHD	42°07′31.98297″ -86°25′45.89033″ 156.0556 m	42°07′31.982767″ -86°25′45.890513″ 156.0871 m	42°07′31.98272″ -86°25′45.89048″ 156.0737 m	42°07′31.98276″ -86°25′45.89053″ 156.0860 m
G317	43°09′42.93110″ -86°13′14.65964″ 155.7060 m	43°09′42.930816″ -86°13′14.659882″ 155.7354 m	43°09′42.93084″ -86°13′146.5980″ 155.7239 m	43°09′42.93082″ -86°13′14.65988″ 155.7352 m
MBYC	42°46′14.12985″ -86°11′55.80754″ 143.2190 m	42°46′14.129585″ -86°11′55.807829″ 143.2488 m	42°46′14.12960″ -86°11′55.80769″ 143.2370 m	42°46′14.12960″ -86°11′55.80778″ 143.2485 m

Table 18: Estimated geodetic coordinates (ϕ, λ, h)

station	RLESS [mm]				BLIMPBE [mm]			
	n	e	u	norm	n	e	u	norm
DET1	8.9	13.0	-37.3	40.5	0.0	0.0	0.0	0.0
MI11	9.6	-1.4	-52.5	53.4	0.0	0.0	0.0	0.0
NLIB	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
SAG1	5.8	5.1	-9.1	11.9	0.0	0.0	0.0	0.0
STB1	13.9	6.2	-30.1	33.7	0.0	0.0	0.0	0.0
WLCI	10.0	7.3	-56.1	57.4	0.0	0.0	0.0	0.0
norm	22.3	17.0	91.0	**95.2**	0.0	0.0	0.0	**0.0**
station	WBLIMPBE [mm]				SCLESS [mm]			
	n	e	u	norm	n	e	u	norm
DET1	1.3	8.5	-19.4	21.2	1.2	3.7	-2.9	4.9
MI11	1.5	-4.4	-34.5	34.8	1.2	-3.7	-8.8	9.6
NLIB	-7.8	-1.8	18.3	20.0	-4.0	-0.1	1.9	4.4
SAG1	-2.3	0.8	8.6	8.9	-2.3	-2.4	9.2	9.8
STB1	5.3	3.0	-12.3	13.7	2.8	1.5	-0.7	3.3
WLCI	2.6	4.1	-37.8	38.1	2.5	2.1	-5.1	6.1
norm	10.2	11.0	59.6	**61.5**	6.2	6.3	14.2	**16.7**

Table 19: Comparison of RLESS, W/BLIMPBE, and SCLESS to a priori coordinates

	BLIMPBE - SCLESS			WBLIMPBE - SCLESS		
Station	n [mm]	e [mm]	u [mm]	n [mm]	e [mm]	u [mm]
DET1	-1.2	-3.7	2.9	0.1	4.8	-16.5
MI11	-1.2	3.7	8.8	0.3	-0.7	-25.7
NLIB	4.0	0.1	-1.9	-3.8	-1.7	16.4
SAG1	2.3	2.4	-9.2	0.0	3.2	-0.6
STB1	-2.8	-1.5	0.7	2.5	1.5	-11.6
WLCI	-2.5	-2.1	5.1	0.1	2.0	-32.7
BEHD	0.4	0.3	1.1	-1.0	1.1	-12.3
G317	-0.2	-0.1	0.2	0.4	1.8	-11.2
MBYC	-0.4	-1.1	0.3	0.0	2.0	-11.6
rms	**2.1**	**2.1**	**4.7**	**1.6**	**2.4**	**17.7**

Table 20: Difference of W/BLIMPBE solution from SCLESS

Table 21 shows the estimated standard deviations, which, consistent with Section 4.3, are shown as the positive square roots of the estimated (parameter) dispersion matrix as expressed in the local geodetic horizon system of each point. Naturally, RLESS yields the larger standard deviations, with its trace of the estimated dispersion matrix being considerably larger than that of the other solutions. While the horizontal standard deviation values computed by W/BLIMPBE and SCLESS for the new stations only differ at the sub–mm level, BLIMBPE and WBLIMPBE are about 1 and 2 mm larger, respectively, than the SCLESS for the standard deviations of the heights of the new points. The BLIMPBE standard deviations for the CORS are all nearly zero. This is in agreement with the "reproducing" characteristic of the selection matrix used in this BLIMPBE solution.

	RLESS			BLIMPBE		
	$\hat{\sigma}_0^2 = 1.02$, $\mathrm{tr}\left(\hat{D}\left\{\hat{\xi}\right\}\right) = 4206 \cdot 10^{-6}\ \mathrm{m}^2$			$\hat{\sigma}_0^2 = 1.28$, $\mathrm{tr}\left(\hat{D}\left\{\hat{\xi}\right\}\right) = 548 \cdot 10^{-6}\ \mathrm{m}^2$		
	$\hat{\sigma}_n$ [mm]	$\hat{\sigma}_e$ [mm]	$\hat{\sigma}_u$ [mm]	$\hat{\sigma}_n$ [mm]	$\hat{\sigma}_e$ [mm]	$\hat{\sigma}_u$ [mm]
DET1	4.9	7.5	22.1	0.0	0.0	0.3
MIL1	4.5	4.6	21.3	0.0	0.0	0.1
NLIB	0.0	0.0	0.0	0.1	0.0	0.0
SAG1	5.7	7.6	21.4	0.0	0.0	0.3
STB1	6.3	5.8	20.4	0.0	0.0	0.2
WLCI	5.4	5.5	26.0	0.0	0.0	0.2
BEHD	3.5	4.7	19.9	2.7	2.2	13.2
G317	5.2	6.0	20.2	2.7	2.1	12.7
MBYC	4.5	5.5	20.4	2.8	2.3	13.2
RMS	**4.8**	**5.7**	**20.3**	**1.3**	**1.3**	**7.5**
	WBLIMPBE			SCLESS		
	$\hat{\sigma}_0^2 = 1.02$, $\mathrm{tr}\left(\hat{D}\left\{\hat{\xi}\right\}\right) = 2171 \cdot 10^{-6}\ \mathrm{m}^2$			$\hat{\sigma}_0^2 = 0.99$, $\mathrm{tr}\left(\hat{D}\left\{\hat{\xi}\right\}\right) = 981 \cdot 10^{-6}\ \mathrm{m}^2$		
	$\hat{\sigma}_n$ [mm]	$\hat{\sigma}_e$ [mm]	$\hat{\sigma}_u$ [mm]	$\hat{\sigma}_n$ [mm]	$\hat{\sigma}_e$ [mm]	$\hat{\sigma}_u$ [mm]
DET1	3.2	4.9	15.4	3.2	3.3	8.2
MIL1	3.3	3.0	15.0	3.3	3.0	8.3
NLIB	3.2	2.7	8.8	3.0	2.1	4.1
SAG1	3.6	4.9	14.6	3.3	3.3	8.0
STB1	4.0	3.7	13.7	3.3	3.1	7.9
WLCI	4.6	3.7	20.2	3.6	3.3	9.0
BEHD	2.7	2.6	13.9	3.2	2.7	12.2
G317	3.1	3.6	14.1	3.3	3.0	11.9
MBYC	2.7	3.1	14.3	3.3	2.8	12.3
RMS	**3.4**	**3.7**	**14.7**	**3.3**	**3.0**	**9.4**

Table 21: Estimated standard deviations (*n*, *e*, *u*) in units of mm

5.3 Summary of New Fiducial Point Adjustments

Only one of the original 23 observed vectors was flagged as an outlier and removed from the data set. The numerical results have confirmed that for the selection matrix chosen in WBLIMPBE, an unbiased adjustment of the observations is achieved (same residuals as generated by Partial MINOLESS). The computations also confirmed the reproducing property of the control points (CORS) for the particular selection matrix used in BLIMPBE and the corresponding (nearly) zero variances. Further, it can be seen from the residuals listed in the respective appendices that this BLIMPBE solution does not belong to the class of LESS.

Finally, the author recommends the adoption of the coordinates computed by the BLIMPBE method (second column of Table 18), using the "standard" selection matrix of (23), for work done on or near the project epoch. This decision is based mainly on the preference for the use of an estimator that generates minimum biases in the new points, when a minimum bias is the best that can be achieved, and that reproduces the control point coordinates. Arguments might also be made for adopting the SCLESS solution instead, subject to further investigations.

CHAPTER 6

CONCLUSIONS

This thesis has proposed and demonstrated a method for *outlier estimation and detection at the GPS–baseline–vector level*. This thesis argues that treating outliers at the baseline–vector level is preferred over the traditional way of testing the vectors component–wise, which leads to decisions about the entire observed baseline vector based upon the hypothesis–test results of the individual components. In fact, the numerical example demonstrated that a contrary decision to flag an observed vector as an outlier can be made if the component–wise method is chosen over the baseline–level method. The baseline–vector level approach also permits use of the correlations between the vector components.

This thesis has also promoted the use of *reliability numbers for correlated observations* for networks of observed GPS baseline vectors, or other types of correlated observations. Instead of the reliability numbers promoted herein, the author has typically seen the use of the so–called redundancy numbers, which may only be of value, and indeed may only lead to correct conclusions when used as an aid in outlier detection processes, if the observations associated with them are truly uncorrelated. Thus, the author encourages geodetic scientists and analysts to use the more theoretically correct method of computing reliability numbers, which does not ignore the correlation between observations.

This thesis has also highlighted the use of the *Best LInear Minimum Partial Bias Estimate* (BLIMPBE) for networks with multiple control points, with two different selection matrices being presented. The solution based on the selection matrix of (23) seems very desirable due to the reproducing property of the control points and the minimization of the biases in the new points; however, it does not factor in the a priori variances of the control points as does SCLESS. Further investigation of alternative selection matrices for BLIMPBE should be a worthwhile study. (Cothren and Schaffrin (1998) have discussed the so–called "reproducing estimator," which also does not change the values of the constrained parameters). Ultimately, whether the BLIMPBE method is chosen over the Adjustment with Stochastic Constraints may depend on whether the scientist or analyst needs to give primacy to the a priori coordinates or to the observations. In some applications one is known with greater certainty than the other. It may well be that the analyst will want to explore the results of both adjustment options before adopting one over the other. With the speed of modern desktop computers, computation duplication is not nearly the concern that it used to be.

Finally, some additional comments are made about conclusions reached for the adjustments carried out in Chapters 4 and 5. The author has already acknowledged that the presumption of only one outlier existing in a data set (as was done herein) may be problematic, and when the final conclusion is that more than one of the observations are candidates for removal, it seems that the conclusion has contradicted the original premise of the test. The author would like to extend this investigation to include tests using the simultaneous outlier–detection routines cited earlier. The author also realizes that some scientific applications (possibly satellite altimetry calibrations) may require better estimates for the establishment of fiducial sites, better than what observations to stations with published CORS coordinates and velocity vectors can yield. In this case, the scientist or researcher might need to call upon the services of an agency that contributes to the ITRF to see if accurate, "current" coordinates are available or can be determined.

Based upon the best access to the ITRF available for this study (i.e., the published CORS information), the author has concluded that heights for new fiducial points can be established with a precision, relative to the CORS network, at a level of ± 1.5 cm ("one–sigma" confidence interval) and even approaching ± 1.0 cm depending upon the adjustment technique. This statement is made regarding observations to a network of stations that vary in distance from 150 km to 430 km from the new stations. The claim is also made based on the field observation procedures outlined in Chapter 3.

END NOTES

1. The National Geodetic Survey, *How CORS Positions and Velocities Were Derived*, published on the NGS web site at http://www.ngs.noaa.gov/CORS/Derivation.html.

2. The International GPS Service, as stated on their web site at http://igscb.jpl.nasa.gov/overview/viewindex.html.

3. The minimum number of days is stated in the reference listed in end note number 1. The number of days used for a particular station is typically listed on the CORS data sheet (see Appendix A).

4. Personal correspondence with Dr. RICHARD SNAY of NGS.

5. The reason for setting the vertical velocities to zero is stated in the reference listed in end note number 1.

6. The use of a symmetrical reflexive generalized inverse to represent a general solution of LESS was discussed by B. SCHAFFRIN in the course GS 762, *Advanced Adjustment Computations* at The Ohio State University in Autumn Quarter of 2000.

7. The invariant properties were shown by B. SCHAFFRIN in the course GS 765, *Analysis and Design of Geodetic Networks* at The Ohio State University in Winter Quarter of 2000.

8. The equivalence of these MINOLESS solutions was shown by B. SCHAFFRIN in the course referred to in end note number 6.

9. The equivalence of these Weighted MINOLESS solutions was shown by B. SCHAFFRIN in the course referred to in end note number 6.

10. CORS data are available from the internet at http://www.ngs.noaa.gov/CORS.

11. The following individuals participated in the collection of GPS data at Lake Michigan: IAN GRENDER, JOHN LIN, DR. MICHAEL PARKE, MOHAMED GADKARIM SALIM, KYLE SNOW, and HONG–ZENG TSENG all of The Ohio State University; and DOUG MARTIN of NGS.

12. Personal correspondence with DAVID ZILKOWSKI of NGS.

13. Documentation for the PAGES program can be found at the NGS web site http://www.ngs.noaa.gov/GRD/GPS/DOC/pages/pages.html.

LIST OF REFERENCES

ADUOL, F.W.O. and B. SCHAFFRIN (1986). *On Outlier Identification in Geodetic Networks Using Principal Component Analysis*, presented at the Conference on Influential Data Analysis, University of Sheffield, England.

Cheng, K., C. Shum, S. Han, Y. Yi, and D. Martin (2001), *Application of GPS–Buoy Water Level Instrument for Radar Altimeter Calibration*, IAG Symposium Series, M. Sideris (eds), Springer–Verlag Berlin Heidelberg, Vol. 123, pp. 367–372.

COTHREN, J. and B. SCHAFFRIN (1998). Towards Optimizing Hierarchical Data Revisions; in: D. Fritsch/M. Englich/M. Sester (eds.), *GIS – Between Visions and Applications*, ISPRS Commission IV Symposium, Stuttgart, Germany, pp. 515–521.

BAARDA, W. (1968). *A Testing Procedure for Use in Geodetic Networks*, Neth. Geod. Com., Publ. on Geodesy, New Series 2, No. 5, Delft Netherlands.

CASPARY, W.F. (1987). *Concepts of Network and Deformation Analysis*, Monograph 11, The University of New South Wales, Kensington, N.S.W. Australia.

GATTI, G. and M. HARWELL (1998). *Advantages of Computer Programs Over Power Charts for the Estimation of Power*, Journal of Statistics Education, v.6, n.3.

GRAFAREND, E. and J. AWANGE (2002). *The Power of the Gauss–Jacobi Combinatorial Algorithm to Detect Outliers and Solve Linear as well as Non–linear Adjustment Problems*, presented at the Weikko A. Heiskanen Symposium in Geodesy, The Ohio State University, Columbus.

GRAFAREND, E. and B. SCHAFFRIN (1993). *Adjustment Computations in Linear Models* (in German), Bibliographical Inst., Mannheim.

IERS (1998). *IERS Technical Note 24*, Central Bureau of IERS – Observatoire de Paris, Paris.

KOCH, K.R., (1999). *Parameter Estimation and Hypothesis Testing in Linear Models*, 2nd Ed., Springer, Berlin.

KUANG, S. (1996). *Geodetic Network Analysis and Optimal Design*, Ann Arbor Press, Chelsea, Michigan.

LEHMER, E. (1944). *Inverse Tables of Probabilities of Errors of the Second Kind*, Annals of Mathematical Statistics, Vol. 15, 388–98.

LEICK, A. (1995). *GPS Satellite Surveying*, 2nd Ed., John Wiley & Sons, Inc, NY.

MIKHAIL, E.M. and F. ACKERMANN (1976). *Observations and Least Squares*, Dun–Donnelley, New York.

NIMA (2000). *Department Of Defense World Geodetic System 1984 – Its Definition and Relationships with Local Geodetic Systems*, TR8350.2, 3rd Ed, National Imagery and Mapping Agency, St Louis, MO.

RAPP, R. (1993). *Geometric Geodesy Part II*, Lecture Notes, Dept. of Geodetic Science and Surveying, The Ohio State University, Columbus, OH.

SCHAFFRIN, B. (1985). Network Design; in: E. Grafarend/F. Sansò (eds.), *Optimization and Design of Geodetic Networks*, Springer–Verlag, Berlin, pp. 548–597.

SCHAFFRIN, B. (1995). *A Generalized Lagrange Functions Approach to Include Fiducial Constraints*, Zeitschrift für Vermessungswessen, Vol. 120, pp. 325–333.

SCHAFFRIN, B. (1997). *Reliability Measures for Correlated Observations*, Journal of Surveying Engineering, Vol. 123, No. 3, pp. 123–137.

SCHAFFRIN, B and H.B. IZ (2002). *BLIMPBE and its Geodetic Applications*; in: *Vistas for Geodesy in the New Millennium*. J. ÁDÁM/K. SCHWARZ (eds.), Springer, Berlin, pp. 377–381.

TIKU, M.L. (1967). *Tables of the Power of the F–Test*, Journal of the American Statistical Association, 62:525–539.

WANG, J. and Y. CHEN (1994). *On the Reliability of Measure of Observations.* Acta Geodaetica et Cartographica Sinica, English Edition, 42–45.

ZILKOWSKI, D., J. D'ONOFRIO, and S. FRAKES (1997). *Guidelines for Establishing GPS–Derived Ellipsoidal Heights (Standards: 2 cm and 5 cm) Version 4.3*", NOAA Technical Memorandum NOS NGS–58, US Dept. of Commerce, National Geodetic Survey, Silver Spring, Maryland.

APPENDIX A

CORS Data Sheets

```
        Antenna Reference Point(ARP): DETROIT 1 CORS ARP
                            PID= AF9501
ITRF96 POSITION (EPOCH 1997.0)
Computed in Mar., 1998 using 47 days of data.
    X =    568024.755 m     latitude    =  42 17 50.45437 N
    Y =  -4690674.635 m     longitude   = 083 05 43.06542 W
    Z =   4270188.820 m     ellipsoid height =    145.045 m

ITRF96 VELOCITY
Computed in Mar., 1998 using 47 days of data.
    VX =  -0.0156 m/yr      northward =  -0.0035 m/yr
    VY =  -0.0043 m/yr      eastward  =  -0.0160 m/yr
    VZ =  -0.0026 m/yr      upward    =   0.0000 m/yr

L1 Phase Center of the current GPS antenna: DETROIT 1 CORS L1 PC C
The ASHTECH GEODETIC III ANTENNA - USCG V antenna (ASH 700829.A1)
was installed on 07/27/95. The L2 phase center is 0.032 m below the L1
phase center.
```

Figure 12: NGS CORS data sheet for station Detroit 1

```
        Antenna Reference Point(ARP): MILWAUKEE 1 CORS ARP
                            PID = AF9485
ITRF96 POSITION (EPOCH 1997.0)
Computed in Mar., 1998 using 52 days of data.
    X =    172136.032 m     latitude    =  43 00 09.13101 N
    Y =  -4668696.644 m     longitude   = 087 53 18.40750 W
    Z =   4327808.348 m     ellipsoid height =    147.377 m

ITRF96 VELOCITY
Computed in Mar., 1998 using 52 days of data.
    VX =  -0.0118 m/yr      northward =  -0.0021 m/yr
    VY =  -0.0019 m/yr      eastward  =  -0.0119 m/yr
    VZ =  -0.0015 m/yr      upward    =   0.0000 m/yr

L1 Phase Center of the current GPS antenna: MILWAUKEE 1 CORS L1 PC C
The ASHTECH GEODETIC III ANTENNA - USCG V antenna (ASH 700829.A1)
was installed on 10/03/95. The L2 phase center is 0.032 m below the L1
phase center.
```

Figure 13: NGS CORS data sheet for station Milwaukee 1

```
        Antenna Reference Point(ARP): NORTH LIBERTY CORS
                           PID = AF9523
ITRF96 POSITION (EPOCH 1997.0)
Published by IERS in Jan., 1998.
    X =   -130934.473 m     latitude    =  41 46 17.72779 N
    Y =  -4762291.774 m     longitude   = 091 34 29.61729 W
    Z =   4226854.704 m     ellipsoid height =    207.096 m

ITRF96 VELOCITY
Published by IERS in Jan., 1998.
    VX =   -.0150 m/yr      northward =  -0.0034 m/yr
    VY =   0.0009 m/yr      eastward  =  -0.0150 m/yr
    VZ =   -.0050 m/yr      upward    =  -0.0037 m/yr

L1 Phase Center of the current GPS antenna: NORTH LIBERTY CORS L1 PC C
The DORNE MARGOLIN T antenna (JPL D/M+crT) was installed on 03/05/93.
The L2 phase center is 0.018 m above the L1 phase center.

                    Monument: NORTH LIBERTY CORS
                            PID = AF9524
                   Inscribed: 4007-S NORTH LIBERTY

ITRF96 POSITION (EPOCH 1997.0)
Published by IERS in Jan., 1998.
    X =   -130934.472 m     latitude    =  41 46 17.72779 N
    Y =  -4762291.729 m     longitude   = 091 34 29.61729 W
    Z =   4226854.663 m     ellipsoid height =    207.035 m
```

Figure 14: NGS CORS data sheet for station North Liberty

```
ITRF96 COORDINATES AT EPOCH 1997.0 AND VELOCITIES
GPS STATIONS

DOMES NB.   SITE NAME       TECH.   ID.
40465M001   NORTH_LIBERTY   GPS     NLIB
        X/Vx          Y/Vy            Z/Vz          Sigmas
                              m/m/y
    -130934.472   -4762291.729    4226854.663   .002    .003    .003
          -.0150          .0009         -.0050  .0005   .0013   .0011
```

Figure 15: Excerpt from *IERS Technical Note 24*

```
        Antenna Reference Point(ARP): SAGINAW 1 CORS ARP
                            PID = AF9510
ITRF96 POSITION (EPOCH 1997.0)
Computed in Mar., 1998 using 56 days of data.
    X =    496374.994 m     latitude    =  43 37 43.11958 N
    Y =  -4597431.512 m     longitude   = 083 50 15.95739 W
    Z =   4378421.351 m     ellipsoid height =    149.223 m

ITRF96 VELOCITY
Computed in Mar., 1998 using 56 days of data.
    VX =  -0.0159 m/yr      northward =   0.0000 m/yr
    VY =  -0.0017 m/yr      eastward  =  -0.0160 m/yr
    VZ =   0.0000 m/yr      upward    =   0.0000 m/yr

L1 Phase Center of the current GPS antenna: SAGINAW 1 CORS L1 PC C
The ASHTECH GEODETIC III ANTENNA - USCG V antenna (ASH 700829.A1)
was installed on 08/24/95. The L2 phase center is 0.032 m below the L1
phase center.
```

Figure 16: NGS CORS data sheet for station Saginaw 1

```
        Antenna Reference Point(ARP): STURGEON BAY 1 CORS ARP
                            PID = PID = AF9553

ITRF96 POSITION (EPOCH 1997.0)
Computed in Mar., 1998 using 48 days of data
    X =    212435.716 m     latitude    =  44 47 43.74825
    Y =  -4528758.901 m     longitude   = 087 18 51.58610
    Z =   4471353.761 m     ellipsoid height =    148.835

ITRF96 VELOCITY
Computed in Mar., 1998 using 48 days of data
    VX =  -0.0164 m/yr      northward =  -0.0038 m/yr
    VY =  -0.0035 m/yr      eastward  =  -0.0165 m/yr
    VZ =  -0.0027 m/yr      upward    =   0.0000 m/yr

L1 Phase Center of the current GPS antenna: STURGEON BAY 1 CORS L1 PC C
The ASHTECH GEODETIC III ANTENNA - USCG V antenna (ASH 700829.A1)
was installed on 01/19/96. The L2 phase center is 0.032 m below the L1
phase center.
```

Figure 17: NGS CORS data sheet for station Sturgeon Bay 1

```
        Antenna Reference Point(ARP): WOLCOTT   CORS ARP
                             PID = AH5611

ITRF96 POSITION (EPOCH 1997.0)
Computed in Dec. 1998 using 11 days of data.
    X =    248645.842 m     latitude    =  40 48 30.26922 N
    Y =  -4828261.314 m     longitude   = 087 03 07.14856 W
    Z =   4146460.096 m     ellipsoid height =    180.424 m

ITRF96 VELOCITY
Predicted with HTDP_2.2 in Dec. 1998.
    VX =  -0.0149 m/yr      northward =  -0.0014 m/yr
    VY =  -0.0017 m/yr      eastward  =  -0.0150 m/yr
    VZ =  -0.0011 m/yr      upward    =   0.0000 m/yr

The GEOD L1/L2 antenna (TRM 22020.00
was installed on 12/01/98. The L2 phase center is 0.006 m below the L1
phase center.
```

Figure 18: NGS CORS data sheet for station Wolcott

APPENDIX B

Data File for CORS Validation Adjustment

```
# Data for CORS height validation testing
# Observed baselines resolved using PAGES
#
# Adjustment type
$RLESS 3
#
# nominal standard errors in n,e,up 0.005, 0.005, 0.010
# standard error in n,e,up for NLIB transformed from values given in
# IERS TN 24 (0.003,0.002,0.003)
# the following are updated coordinates using the NGS published
# velocity vectors
#
# CORS coordinates in X,Y,Z (1999.321 epoch) and standard deviations in
n,e,up
$XYZ DET1  568024.7189  -4690674.6449  4270188.8140  0.005  0.005  0.01
$XYZ MIL1  172136.0047  -4668696.6484  4327808.3445  0.005  0.005  0.01
$XYZ NLIB -130934.5067  -4762291.7269  4226854.6514  0.003  0.002 0.003
$XYZ SAG1  496374.9572  -4597431.5159  4378421.3510  0.005  0.005  0.01
$XYZ STB1  212435.6781  -4528758.9091  4471353.7548  0.005  0.005  0.01
$XYZ WLCI  248645.8076  -4828261.3179  4146460.0935  0.005  0.005  0.01
#
$BEGOBS
#
# description of data record:
# obs type code; obs from to; dX; dY; dZ;
# var(dX); covar(dX,dY); var(dY); covar(dX,dZ); covar(dY,dZ); var(dZ)
# <-lower triagular covariance matrix
#
# Data Set 1
# DOY 064
$GPS NLIB  MIL1  303070.4873   93595.0868  100953.6848
1.6000000000e-07 -5.0645520000e-09  3.2400000000e-06  4.4407872000e-08
-2.7129208320e-06 2.5600000000e-06
$GPS NLIB  STB1  343370.1879  233532.8082  244499.1146
1.6000000000e-07  4.3582320000e-08  3.2400000000e-06  6.2520768000e-08
-2.6486228160e-06 2.5600000000e-06
$GPS NLIB  SAG1  627309.4588  164860.1847  151566.7236
2.5000000000e-07  2.8482750000e-08  3.2400000000e-06  4.2685045000e-08
-2.8781445060e-06 2.8900000000e-06
$GPS NLIB  DET1  698959.2192   71617.0870   43334.1768
3.6000000000e-07 -1.2295584000e-08  3.6100000000e-06  3.1400640000e-08
-2.8832256800e-06 2.5600000000e-06
```

```
# DOY 065
$GPS WLCI  STB1  -36210.1220  299502.4067  324893.6501
4.0000000000e-08 -7.7833280000e-08  1.9600000000e-06  5.4380256000e-08
-1.4953186080e-06 1.4400000000e-06
$GPS MIL1  STB1   40299.6816  139937.7255  143545.4233
4.0000000000e-08 -4.6709256000e-08  1.4400000000e-06  5.1864216000e-08
-1.3388257440e-06 1.4400000000e-06
$GPS MIL1  DET1  395888.7290  -21978.0055  -57619.5150
1.6000000000e-07 -1.4391826400e-07  1.9600000000e-06  1.0552622400e-07
-1.6006546080e-06 1.4400000000e-06
# DOY 066
$GPS STB1  SAG1  283939.2827  -68672.6080  -92932.4050
1.6000000000e-07 -1.7677566000e-07  2.2500000000e-06  1.3086847200e-07
-1.9746165600e-06 1.9600000000e-06
$GPS STB1  DET1  355589.0576 -161915.7376 -201164.9324
2.5000000000e-07 -2.4818407500e-07  2.2500000000e-06  1.2399317000e-07
-1.9279673700e-06 1.9600000000e-06
# DOY 067
$GPS SAG1  MIL1 -324238.9628  -71265.1285  -50613.0072
1.6000000000e-07 -1.3257196800e-07  1.4400000000e-06  1.4718576400e-07
-1.2439616640e-06 1.2100000000e-06
$GPS SAG1  DET1   71649.7642  -93243.1321 -108232.5245
9.0000000000e-08 -1.5966182700e-07  1.6900000000e-06  1.3463320200e-07
-1.3262939690e-06 1.2100000000e-06
# DOY 068
$GPS WLCI  NLIB -379580.3089   65969.5692   80394.5595
1.6000000000e-07  4.9139608000e-08  1.9600000000e-06 -7.0045040000e-08
-1.7078263020e-06 1.6900000000e-06
$GPS WLCI  MIL1  -76509.7968  159564.6675  181348.2377
4.0000000000e-08 -3.1087034000e-08  1.2100000000e-06  2.1052280000e-08
-1.0184101400e-06 1.0000000000e-06
$GPS WLCI  SAG1  247729.1634  230829.8005  231961.2424
9.0000000000e-08 -9.2176020000e-08  1.4400000000e-06  1.0280511000e-07
-1.1058626400e-06 1.0000000000e-06
$GPS WLCI  DET1  319378.9265  137586.6704  123728.7143
9.0000000000e-08 -9.7659720000e-08  1.4400000000e-06  1.0050243000e-07
-1.1195924400e-06 1.0000000000e-06
#
# Data Set 2
# DOY 079
$GPS NLIB  MIL1  303070.5095   93595.0682  100953.6897
1.6000000000e-07  1.8379840000e-09  3.6100000000e-06  2.1211308000e-08
-3.0461057640e-06 2.8900000000e-06
$GPS NLIB  STB1  343370.1774  233532.7983  244499.1267
1.6000000000e-07  8.0924116000e-08  3.6100000000e-06  1.3284864000e-08
-3.1643276400e-06 3.2400000000e-06
$GPS NLIB  SAG1  627309.4586  164860.2095  151566.6960
3.6000000000e-07  8.3331738000e-08  4.4100000000e-06  1.6241472000e-08
-3.5352612540e-06 3.2400000000e-06
$GPS NLIB  DET1  698959.2059   71617.0780   43334.1791
3.6000000000e-07  3.7522548000e-08  4.4100000000e-06 -1.5540366000e-08
-3.7877895930e-06 3.6100000000e-06
# DOY 080
$GPS DET1  MIL1 -395888.7275   21977.9926   57619.5322
```

```
2.5000000000e-07 -2.5716393000e-07  2.8900000000e-06  1.6463520000e-07
-2.4044462550e-06 2.2500000000e-06
# DOY 081
$GPS STB1  MIL1  -40299.6831 -139937.7257 -143545.4248
9.0000000000e-08 -5.6522928000e-08  1.9600000000e-06  6.4589304000e-08
-1.6909913020e-06 1.6900000000e-06
$GPS STB1  SAG1  283939.2848  -68672.6097  -92932.4075
9.0000000000e-08 -1.4471078700e-07  1.6900000000e-06  1.1028700800e-07
-1.5886196040e-06 1.6900000000e-06
$GPS STB1  DET1  355589.0432 -161915.7338 -201164.9447
1.6000000000e-07 -1.9381936000e-07  1.9600000000e-06  1.0652548400e-07
-1.6880652880e-06 1.6900000000e-06
# DOY 082
$GPS WLCI  NLIB -379580.3079   65969.5812   80394.5528
9.0000000000e-08  -5.2675140000e-09 1.9600000000e-06  2.3285880000e-09
-1.5938727840e-06 1.4400000000e-06
$GPS WLCI  MIL1  -76509.8036  159564.6650  181348.2391
4.0000000000e-08 -5.7735696000e-08  1.4400000000e-06  4.6844006000e-08
-1.2333800160e-06 1.2100000000e-06
$GPS WLCI  STB1  -36210.1200  299502.4000  324893.6528
9.0000000000e-08 -3.7628604000e-08  1.6900000000e-06  4.0373208000e-08
-1.4250367560e-06 1.4400000000e-06
$GPS WLCI  SAG1  247729.1604  230829.7950  231961.2460
9.0000000000e-08 -1.0248220800e-07  1.4400000000e-06  1.2835036500e-07
-1.2224098920e-06 1.2100000000e-06
$GPS WLCI  DET1  319378.9210  137586.6583  123728.7187
9.0000000000e-08 -1.2816870300e-07  1.6900000000e-06  1.2581319300e-07
-1.3462579130e-06 1.2100000000e-06
# DOY 083
$GPS SAG1  MIL1 -324238.9627  -71265.1298  -50613.0076
1.6000000000e-07 -1.4546688800e-07  1.9600000000e-06  1.6109860000e-07
-1.7206199920e-06 1.6900000000e-06
$GPS SAG1  DET1   71649.7560  -93243.1265 -108232.5321
9.0000000000e-08 -2.3168502000e-07  1.9600000000e-06  1.9676389200e-07
-1.6999389680e-06 1.6900000000e-06
#
# Data Set 3
# DOY 131
$GPS SAG1  MIL1 -324238.9561  -71265.1293  -50613.0039
2.5000000000e-07 -1.7593350000e-07  2.2500000000e-06  2.1520142000e-07
-1.9788363000e-06 1.9600000000e-06
$GPS SAG1  DET1   71649.7569  -93243.1248 -108232.5287
9.0000000000e-08 -2.4079855800e-07  1.9600000000e-06  1.9997729700e-07
-1.7024309120e-06 1.6900000000e-06
# DOY 132
$GPS WLCI  MIL1  -76509.8078  159564.6742  181348.2370
9.0000000000e-08 -1.4289067200e-07  2.5600000000e-06  1.1302569600e-07
-2.0721341760e-06 1.9600000000e-06
$GPS WLCI  STB1  -36210.1189  299502.4108  324893.6546
1.6000000000e-07 -1.9592568000e-07  2.8900000000e-06  1.5777210000e-07
-2.2607999100e-06 2.2500000000e-06
$GPS WLCI  SAG1  247729.1551  230829.8032  231961.2481
1.6000000000e-07 -1.8938727600e-07  2.8900000000e-06  2.4126618000e-07
-2.3193884550e-06 2.2500000000e-06
$GPS WLCI  DET1  319378.9159  137586.6688  123728.7271
```

```
1.6000000000e-07 -1.5015086800e-07  2.8900000000e-06  1.9830762000e-07
-2.3827432050e-06 2.2500000000e-06
# DOY 133
$GPS DET1  MIL1 -395888.7206   21978.0014   57619.5230
3.6000000000e-07 -3.5597242800e-07  3.2400000000e-06  2.8532851200e-07
-2.7329063040e-06 2.5600000000e-06
# DOY 134
$GPS NLIB  MIL1  303070.5033   93595.0934  100953.6803
9.0000000000e-08  3.9536955000e-08  2.2500000000e-06 -3.4111740000e-09
-1.8535740600e-06 1.6900000000e-06
$GPS NLIB  STB1  343370.1892  233532.8166  244499.1077
1.6000000000e-07  7.1691420000e-08  2.2500000000e-06  1.2568024000e-08
-1.9596895500e-06 1.9600000000e-06
$GPS NLIB  WLCI  379580.3079  -65969.5853  -80394.5519
1.6000000000e-07 -1.0675648000e-08  2.5600000000e-06 -1.2579784000e-08
-2.1218162560e-06 1.9600000000e-06
$GPS NLIB  SAG1   627309.4673  164860.2071  151566.6992
1.6000000000e-07  7.9700608000e-08  2.5600000000e-06 -1.0493616000e-08
-2.1185758720e-06 1.9600000000e-06
$GPS NLIB  DET1  698959.2342   71617.0964   43334.1605
2.5000000000e-07  5.1904320000e-08  2.5600000000e-06 -2.5639740000e-08
-2.1376425280e-06 1.9600000000e-06
# DOY 135
$GPS STB1  MIL1  -40299.6839 -139937.7382 -143545.4220
4.0000000000e-08 -6.2861036000e-08  1.9600000000e-06  6.9019468000e-08
-1.8266721760e-06 1.9600000000e-06
$GPS STB1  SAG1  283939.2732  -68672.6091  -92932.4100
1.6000000000e-07 -2.1714600000e-07  2.2500000000e-06  1.6058100800e-07
-1.9860199800e-06 1.9600000000e-06
$GPS STB1  DET1  355589.0380 -161915.7331 -201164.9407
1.6000000000e-07 -2.1795834000e-07  2.2500000000e-06  1.2143768000e-07
-1.9525413600e-06 1.9600000000e-06
```

APPENDIX C

Data File for New Fiducial Points Adjustment

```
# Data for new fiducial points survey
# Observed baselines resolved using PAGES
#
# Adjustment type
$RLESS 3
#
# CORS coordinates in X,Y,Z (1999.442 epoch) and std dev in n,e,up
$XYZ DET1  568024.7169  -4690674.6455  4270188.8137  0.005  0.005  0.01
$XYZ MIL1  172136.0032  -4668696.6486  4327808.3443  0.005  0.005  0.01
$XYZ NLIB -130934.5086  -4762291.7268 4226854.6508 .00418 .00235 .00422
$XYZ SAG1  496374.9552  -4597431.5162  4378421.3510  0.005  0.005  0.01
$XYZ STB1  212435.6760  -4528758.9095  4471353.7544  0.005  0.005  0.01
$XYZ WLCI  248645.8056  -4828261.3182  4146460.0933  0.005  0.005  0.01
#
# a priori coordinates for new fiducial points
$XYZ G317  307138.848   -4649646.701   4340747.247    &      &     &
$XYZ BEHD  295059.735   -4728575.241   4256061.833    &      &     &
$XYZ MBYC  310880.092   -4679085.806   4308925.673    &      &     &
#
# stations with centering errors (name horizontal vertical)
$CENTER_ERR  G317  0.003  0.000
$CENTER_ERR  BEHD  0.003  0.000
$CENTER_ERR  MBYC  0.003  0.000
#
$BEGOBS
#
# description of data record:
# on/off code; obs from to; dX; dY; dZ;
# var(dX); covar(dX,dY); var(dY); covar(dX,dZ); covar(dY,dZ); var(dZ)
#
# DOY 160
$GPS MBYC  G317   -3741.2376   29439.0952   31821.5696
1.6000000000e-07 -3.2477453600e-07 5.2900000000e-06 2.8992961200e-07
-4.5752170170e-06 4.4100000000e-06
$GPS SAG1  G317 -189236.1424  -52215.1555  -37674.1125
2.5000000000e-07 -4.8443945500e-07 5.2900000000e-06 4.8305323000e-07
-4.7892616640e-06 4.8400000000e-06
$GPS DET1  MBYC -257144.6615   11588.8525   38736.8600
2.5000000000e-07 -5.3013500000e-07 6.2500000000e-06 4.6678960000e-07
-5.4494664750e-06 5.2900000000e-06
$GPS BEHD  MBYC   15820.3572   49489.4463   52863.8501
1.6000000000e-07 -3.6881330000e-07 6.2500000000e-06 3.1879716000e-07
```

```
-5.2199576000e-06 4.8400000000e-06
$GPS NLIB  BEHD  425994.2056   33716.5182   29207.1549
3.6000000000e-07 1.1911430400e-07 1.0240000000e-05 -1.3411759200e-07
-8.5999952640e-06 7.8400000000e-06
$GPS MIL1  BEHD  122923.6979  -59878.6067  -71746.5038
1.6000000000e-07 -2.3454748800e-07 5.7600000000e-06 1.6015742400e-07
-4.7683596240e-06 4.4100000000e-06
$GPS G317  STB1  -94703.1397  120887.7846  130606.5082
1.6000000000e-07 -3.2950793600e-07 5.2900000000e-06 2.7098913600e-07
-4.7494657760e-06 4.8400000000e-06
# DOY 161
$GPS NLIB  BEHD  425994.1987   33716.5441   29207.1454
1.0240000000e-05 4.1382374400e-06 2.0250000000e-05 -1.9865111040e-06
-1.6156820430e-05 1.4440000000e-05
$GPS MIL1  BEHD  122923.6872  -59878.5719  -71746.5240
1.0000000000e-06 -7.5661111000e-07 9.6100000000e-06 -1.8105108000e-07
-7.7596604680e-06 7.8400000000e-06
$GPS MBYC  BEHD  -15820.3673  -49489.4261  -52863.8515
2.5000000000e-07 -5.9511216000e-07 1.0240000000e-05 5.2836927000e-07
-8.4754472000e-06 8.4100000000e-06
$GPS G317  MBYC    3741.2373  -29439.1032  -31821.5683
2.5000000000e-07 -6.1664549000e-07 9.6100000000e-06 4.9177953500e-07
-8.3942362960e-06 8.4100000000e-06
$GPS SAG1  G317 -189236.1416  -52215.1569  -37674.1254
2.2500000000e-06 -3.8544000000e-07 1.0240000000e-05 1.1512840500e-06
-8.5739924480e-06 8.4100000000e-06
$GPS DET1  MBYC -257144.6734   11588.8359   38736.8742
3.6100000000e-06 -1.2204525550e-06 1.2250000000e-05 4.0914569600e-07
-1.0462760000e-05 1.0240000000e-05
$GPS STB1  G317   94703.1393 -120887.7727 -130606.5306
6.4000000000e-07 -9.8079486400e-07 9.6100000000e-06 -2.7710404000e-07
-7.4499324310e-06 9.6100000000e-06
$GPS BEHD  WLCI  -46413.8842  -99686.0721 -109601.7420
4.9000000000e-07 -5.6362261200e-07 1.4440000000e-05 9.0787365900e-07
-1.0561932876e-05 1.0890000000e-05
# DOY 162
$GPS NLIB  BEHD  425994.2335   33716.5709   29207.1412
1.0240000000e-05 3.7531000320e-06 1.9360000000e-05 -1.3934442240e-06
-1.4923440576e-05 1.2960000000e-05
$GPS MIL1  BEHD  122923.7390  -59878.6232  -71746.5195
1.2100000000e-06 -1.3677123900e-06 1.1560000000e-05 2.4549857200e-07
-8.7115002980e-06 8.4100000000e-06
$GPS MBYC  BEHD  -15820.3763  -49489.4339  -52863.8539
2.5000000000e-07 -5.3892870000e-07 9.0000000000e-06 5.3268975000e-07
-7.3769648400e-06 7.2900000000e-06
$GPS G317  MBYC    3741.2415  -29439.1022  -31821.5851
2.5000000000e-07 -5.5178778500e-07 8.4100000000e-06 4.6384065000e-07
-7.0073691220e-06 6.7600000000e-06
$GPS SAG1  G317 -189236.1477  -52215.1632  -37674.1318
2.2500000000e-06 -7.8022395000e-07 1.0890000000e-05 1.4010932400e-06
-8.8124148750e-06 8.4100000000e-06
$GPS DET1  MBYC -257144.6977   11588.8736   38736.8391
4.0000000000e-06 -1.4652187200e-06 1.2960000000e-05 6.3041724000e-07
-1.0512817092e-05 9.6100000000e-06
$GPS STB1  G317   94703.1446 -120887.7869 -130606.5361
```

```
8.1000000000e-07 -6.8160015000e-07 9.0000000000e-06 -5.2916895000e-07
-6.9304932000e-06 9.0000000000e-06
$GPS BEHD  WLCI  -46413.8783  -99686.1022 -109601.7399
6.4000000000e-07 -1.7321676800e-07 1.4440000000e-05 5.9135872000e-07
-1.0363120448e-05 1.0240000000e-05
```

APPENDIX D

RLESS for CORS Validation, 45 Observed Baseline Vectors

```
The 3x3 block diagonal covariance matrix is replaced by a full
(session) matrix
Adjustment type: (RLESS) Restricted Least-Squares Solution
Ellipsoid: WGS84
Units: dms, meters

No of observations                      :  135
Rank of A                               : - 15
                                          ----
System redundancy                       :  120

Estimated parameters: Cartesian (meters)
Name          X               Y               Z
DET1     568024.7204   -4690674.6401    4270188.8175
MIL1     172135.9968   -4668696.6404    4327808.3376
NLIB    -130934.5067   -4762291.7269    4226854.6514
SAG1     496374.9593   -4597431.5138    4378421.3477
STB1     212435.6792   -4528758.9083    4471353.7567
WLCI     248645.7991   -4828261.3107    4146460.1022

Estimated parameters: geodetic (ddd.mmsssssss)
Name      latitude        longitude      height
DET1    42.175045429   -83.054306694    145.0434
MIL1    43.000913087   -87.531840903    147.3668
NLIB    41.461772752   -91.342961878    207.0266
SAG1    43.374311954   -83.501595894    149.2196
STB1    44.474374803   -87.185158779    148.8363
WLCI    40.483026948   -87.030715037    180.4233


Trace of estimated dispersion matrix: 0.000576
Estimated reference variance: 145.9125

Estimated standard errors (scaled by sqrt estimated reference variance)
Name    std(X)   std(Y)   std(Z)    std(n)  std(e)   std(up)
          m        m        m         m       m        m
DET1    0.0021   0.0077   0.0068   0.0016   0.0024   0.0101
MIL1    0.0017   0.0076   0.0067   0.0016   0.0018   0.0099
NLIB    0.0000   0.0000   0.0000   0.0001   0.0000   0.0000
SAG1    0.0020   0.0078   0.0069   0.0017   0.0023   0.0102
STB1    0.0018   0.0080   0.0072   0.0019   0.0019   0.0106
WLCI    0.0019   0.0084   0.0074   0.0019   0.0019   0.0110
```

```
Observation Estimates
Obs#       From-To
Obs#    dX/dY/dZ      Obs.       Adjusted    Obs.       Stu.  Trad. Std.
           Obs.       Error         Obs.    Std. Dev.   Res.  Red # Rel #
Vec01: NLIB -> MIL1
  1   303070.4873   -0.0162    303070.503  0.00171  -3.580  0.90  0.93*
  2    93595.0868    0.0003     93595.087  0.00755   0.014  0.92  0.95
  3   100953.6848   -0.0014    100953.686  0.00669  -0.079  0.92  0.94
Vec02: NLIB -> STB1
  4   343370.1879    0.0020    343370.186  0.00183   0.449  0.87  0.90
  5   233532.8082   -0.0104    233532.819  0.00797  -0.516  0.96  0.92
  6   244499.1146    0.0093    244499.105  0.00723   0.520  0.84  0.90
Vec03: NLIB -> SAG1
  7   627309.4588   -0.0072    627309.466  0.00205  -1.266  0.92  0.95
  8   164860.1847   -0.0284    164860.213  0.00776  -1.396  0.82  0.93
  9   151566.7236    0.0273    151566.696  0.00688   1.412  1.01  0.94
Vec04: NLIB -> DET1
 10   698959.2192   -0.0079    698959.227  0.00214  -1.136  0.97  0.96
 11    71617.0870    0.0002     71617.087  0.00774   0.010  0.99  0.94
 12    43334.1768    0.0107     43334.166  0.00679   0.594  0.85  0.93
Vec05: WLCI -> STB1
 13   -36210.1220   -0.0020    -36210.120  0.00132  -1.010  0.70  0.74
 14   299502.4067    0.0043    299502.402  0.00709   0.280  0.91  0.81
 15   324893.6501   -0.0044    324893.655  0.00629  -0.339  0.70  0.80
Vec06: MIL1 -> STB1
 16    40299.6816   -0.0008     40299.682  0.00111  -0.377  0.80  0.82
 17   139937.7255   -0.0066    139937.732  0.00547  -0.493  0.71  0.83
 18   143545.4233    0.0042    143545.419  0.00532   0.315  1.01  0.84
Vec07: MIL1 -> DET1
 19   395888.7290    0.0054    395888.724  0.00136   1.169  0.92  0.88
 20   -21978.0055   -0.0058    -21978.000  0.00485  -0.356  0.98  0.89
 21   -57619.5150    0.0052    -57619.520  0.00435   0.374  0.84  0.88
Vec08: STB1 -> SAG1
 22   283939.2827    0.0026    283939.280  0.00137   0.560  0.92  0.93
 23   -68672.6080   -0.0024    -68672.606  0.00562  -0.140  0.93  0.91
 24   -92932.4050    0.0040    -92932.409  0.00546   0.251  0.88  0.91
Vec09: STB1 -> DET1
 25   355589.0576    0.0164    355589.041  0.00145   2.801  0.95  0.95*
 26  -161915.7376   -0.0057   -161915.732  0.00557  -0.333  0.92  0.92
 27  -201164.9324    0.0068   -201164.939  0.00531   0.426  0.92  0.92
Vec10: SAG1 -> MIL1
 28  -324238.9628   -0.0003   -324238.963  0.00127  -0.062  0.93  0.93
 29   -71265.1285   -0.0020    -71265.127  0.00497  -0.144  0.90  0.89
 30   -50613.0072    0.0028    -50613.010  0.00462   0.228  0.87  0.89
Vec11: SAG1 -> DET1
 31    71649.7642    0.0031     71649.761  0.00118   0.914  0.90  0.90
 32   -93243.1321   -0.0058    -93243.126  0.00495  -0.391  0.97  0.92
 33  -108232.5245    0.0057   -108232.530  0.00450   0.457  0.84  0.91
Vec12: WLCI -> NLIB
 34  -379580.3089   -0.0031   -379580.306  0.00187  -0.685  0.85  0.85
 35    65969.5692   -0.0146     65969.584  0.00845  -0.994  0.64  0.80
 36    80394.5595    0.0103     80394.549  0.00738   0.740  0.94  0.82
Vec13: WLCI -> MIL1
 37   -76509.7968    0.0056    -76509.802  0.00115   2.620  0.80  0.82*
 38   159564.6675   -0.0028    159564.670  0.00648  -0.240  0.71  0.86
```

```
 39   181348.2377    0.0022    181348.235  0.00561   0.208  0.93  0.87
Vec14: WLCI -> SAG1
 40   247729.1634    0.0033    247729.160  0.00141   0.974  0.88  0.90
 41   230829.8005    0.0037    230829.797  0.00676   0.287  0.98  0.86
 42   231961.2424   -0.0031    231961.246  0.00582  -0.295  0.68  0.84
Vec15: WLCI -> DET1
 43   319378.9265    0.0053    319378.921  0.00144   1.590  0.86  0.88
 44   137586.6704   -0.0001    137586.671  0.00668  -0.011  0.90  0.88
 45   123728.7143   -0.0010    123728.715  0.00562  -0.094  0.79  0.87
Vec16: NLIB -> MIL1
 46   303070.5095    0.0060    303070.503  0.00171   1.331  0.89  0.93
 47    93595.0682   -0.0183     93595.087  0.00755  -0.845  0.93  0.95
 48   100953.6897    0.0035    100953.686  0.00669   0.179  0.92  0.95
Vec17: NLIB -> STB1
 49   343370.1774   -0.0085    343370.186  0.00183  -1.898  0.86  0.90
 50   233532.7983   -0.0203    233532.819  0.00797  -0.945  0.88  0.92
 51   244499.1267    0.0214    244499.105  0.00723   1.044  0.95  0.93
Vec18: NLIB -> SAG1
 52   627309.4586   -0.0074    627309.466  0.00205  -1.064  0.97  0.96
 53   164860.2095   -0.0036    164860.213  0.00776  -0.147  0.98  0.96
 54   151566.6960   -0.0003    151566.696  0.00688  -0.013  0.90  0.95
Vec19: NLIB -> DET1
 55   698959.2059   -0.0212    698959.227  0.00214  -3.057  0.96  0.96*
 56    71617.0780   -0.0088     71617.087  0.00774  -0.363  0.91  0.95
 57    43334.1791    0.0130     43334.166  0.00679   0.595  0.97  0.95
Vec20: DET1 -> MIL1
 58  -395888.7275   -0.0039   -395888.724  0.00136  -0.666  0.95  0.94
 59    21977.9926   -0.0071     21978.000  0.00485  -0.357  0.95  0.94
 60    57619.5322    0.0120     57619.520  0.00435   0.684  0.93  0.94
Vec21: STB1 -> MIL1
 61   -40299.6831   -0.0007    -40299.682  0.00111  -0.200  0.92  0.92
 62  -139937.7257    0.0064   -139937.732  0.00547   0.401  0.95  0.92
 63  -143545.4248   -0.0057   -143545.419  0.00532  -0.389  0.87  0.92
Vec22: STB1 -> SAG1
 64   283939.2848    0.0047    283939.280  0.00137   1.399  0.83  0.85
 65   -68672.6097   -0.0041    -68672.606  0.00562  -0.281  0.82  0.89
 66   -92932.4075    0.0015    -92932.409  0.00546   0.103  0.95  0.90
Vec23: STB1 -> DET1
 67   355589.0432    0.0020    355589.041  0.00145   0.440  0.93  0.92
 68  -161915.7338   -0.0019   -161915.732  0.00557  -0.122  0.92  0.92
 69  -201164.9447   -0.0055   -201164.939  0.00531  -0.370  0.89  0.91
Vec24: WLCI -> NLIB
 70  -379580.3079   -0.0021   -379580.306  0.00187  -0.662  0.73  0.75
 71    65969.5812   -0.0026     65969.584  0.00845  -0.175  0.83  0.74
 72    80394.5528    0.0036     80394.549  0.00738   0.285  0.64  0.73
Vec25: WLCI -> MIL1
 73   -76509.8036   -0.0012    -76509.802  0.00115  -0.582  0.81  0.82
 74   159564.6650   -0.0053    159564.670  0.00648  -0.407  0.77  0.87
 75   181348.2391    0.0036    181348.235  0.00561   0.301  0.94  0.88
Vec26: WLCI -> STB1
 76   -36210.1200   -0.0000    -36210.120  0.00132  -0.013  0.90  0.91
 77   299502.4000   -0.0024    299502.402  0.00709  -0.171  0.82  0.84
 78   324893.6528   -0.0017    324893.655  0.00629  -0.133  0.86  0.84
Vec27: WLCI -> SAG1
 79   247729.1604    0.0003    247729.160  0.00141   0.076  0.88  0.89
```

```
 80   230829.7950   -0.0018    230829.797  0.00676  -0.142  0.72  0.85
 81   231961.2460    0.0005    231961.246  0.00582   0.040  0.94  0.86
Vec28: WLCI -> DET1
 82   319378.9210   -0.0002    319378.921  0.00144  -0.065  0.85  0.87
 83   137586.6583   -0.0122    137586.671  0.00668  -0.862  0.93  0.88
 84   123728.7187    0.0034    123728.715  0.00562   0.282  0.80  0.88
Vec29 SAG1 -> MIL1
 85  -324238.9627   -0.0002   -324238.963  0.00127  -0.040  0.93  0.93
 86   -71265.1298   -0.0033    -71265.127  0.00497  -0.202  0.92  0.92
 87   -50613.0076    0.0024    -50613.010  0.00462   0.163  0.91  0.92
Vec30: SAG1 -> DET1
 88    71649.7560   -0.0051     71649.761  0.00118  -1.479  0.89  0.89
 89   -93243.1265   -0.0002    -93243.126  0.00495  -0.014  0.89  0.92
 90  -108232.5321   -0.0019   -108232.530  0.00450  -0.125  0.95  0.93
Vec31: SAG1 -> MIL1
 91  -324238.9561    0.0064   -324238.963  0.00127   1.086  0.96  0.95
 92   -71265.1293   -0.0028    -71265.127  0.00497  -0.158  0.93  0.93
 93   -50613.0039    0.0061    -50613.010  0.00462   0.377  0.92  0.93
Vec32: SAG1 -> DET1
 94    71649.7569   -0.0042     71649.761  0.00118  -1.216  0.88  0.88
 95   -93243.1248    0.0015    -93243.126  0.00495   0.091  0.89  0.91
 96  -108232.5287    0.0015   -108232.530  0.00450   0.101  0.94  0.92
Vec33: WLCI -> MIL1
 97   -76509.8078   -0.0054    -76509.802  0.00115  -1.582  0.91  0.92
 98   159564.6742    0.0039    159564.670  0.00648   0.215  0.92  0.93
 99   181348.2370    0.0015    181348.235  0.00561   0.095  0.90  0.93
Vec34: WLCI -> STB1
100   -36210.1189    0.0011    -36210.120  0.00132   0.227  0.94  0.94
101   299502.4108    0.0084    299502.402  0.00709   0.436  0.92  0.92
102   324893.6546    0.0001    324893.655  0.00629   0.004  0.89  0.92
Vec35: WLCI -> SAG1
103   247729.1551   -0.0050    247729.160  0.00141  -1.092  0.93  0.93
104   230829.8032    0.0064    230829.797  0.00676   0.329  0.92  0.94
105   231961.2481    0.0026    231961.246  0.00582   0.150  0.93  0.94
Vec36: WLCI -> DET1
106   319378.9159   -0.0053    319378.921  0.00144  -1.153  0.92  0.92
107   137586.6688   -0.0017    137586.671  0.00668  -0.090  0.88  0.93
108   123728.7271    0.0118    123728.715  0.00562   0.685  0.97  0.93
Vec37: DET1 -> MIL1
109  -395888.7206    0.0030   -395888.724  0.00136   0.419  0.96  0.96
110    21978.0014    0.0017     21978.000  0.00485   0.079  0.95  0.95
111    57619.5230    0.0028     57619.520  0.00435   0.150  0.94  0.95
Vec38: NLIB -> MIL1
112   303070.5033   -0.0002    303070.503  0.00171  -0.057  0.78  0.85
113    93595.0934    0.0069     93595.087  0.00755   0.418  0.97  0.90
114   100953.6803   -0.0059    100953.686  0.00669  -0.418  0.78  0.89
Vec39: NLIB -> STB1
115   343370.1892    0.0033    343370.186  0.00183   0.739  0.90  0.91
116   233532.8166   -0.0020    233532.819  0.00797  -0.125  0.77  0.85
117   244499.1077    0.0024    244499.105  0.00723   0.158  0.92  0.86
Vec40: NLIB -> WLCI
118   379580.3079    0.0021    379580.306  0.00187   0.461  0.90  0.90
119   -65969.5853   -0.0015    -65969.584  0.00845  -0.088  0.80  0.84
120   -80394.5519   -0.0027    -80394.549  0.00738  -0.174  0.87  0.84
Vec41: NLIB -> SAG1
```

```
121   627309.4673    0.0013    627309.466  0.00205   0.298  0.84  0.88
122   164860.2071   -0.0060    164860.213  0.00776  -0.336  0.95  0.91
123   151566.6992    0.0029    151566.696  0.00688   0.189  0.85  0.90
Vec42: NLIB -> DET1
124   698959.2342    0.0071    698959.227  0.00214   1.264  0.96  0.93
125    71617.0964    0.0096     71617.087  0.00774   0.543  0.88  0.91
126    43334.1605   -0.0056     43334.166  0.00679  -0.359  0.90  0.91
Vec43: STB1 -> MIL1
127   -40299.6839   -0.0015    -40299.682  0.00111  -0.695  0.79  0.80
128  -139937.7382   -0.0061   -139937.732  0.00547  -0.380  0.84  0.91
129  -143545.4220   -0.0029   -143545.419  0.00532  -0.183  0.99  0.92
Vec44: STB1 -> SAG1
130   283939.2732   -0.0069    283939.280  0.00137  -1.490  0.93  0.92
131   -68672.6091   -0.0035    -68672.606  0.00562  -0.204  0.96  0.91
132   -92932.4100   -0.0010    -92932.409  0.00546  -0.062  0.84  0.90
Vec45: STB1 -> DET1
133   355589.0380   -0.0032    355589.041  0.00145  -0.688  0.91  0.91
134  -161915.7331   -0.0012   -161915.732  0.00557  -0.072  0.93  0.92
135  -201164.9407   -0.0015   -201164.939  0.00531  -0.091  0.90  0.92

Sum of traditional redundancy numbers   =  120.00
Sum of standardized reliability numbers =  121.26
```

APPENDIX E

WMINOLESS for CORS Validation, 41 Observed Baseline Vectors

```
GPS observation variances and covariances scaled by 96.0 beginning at
observation 1.
The 3x3 block diagonal covariance matrix is replaced by a full
(session) matrix.
Adjustment type: Weighted Minimum Norm Least-Squares Solution
Units: dms, meters

No of observations    :  123
Rank of A             : - 15
                        ----
System redundancy     :  108

Adjustment PASSED the Chi Square test at the 95% Confidence Level
Lower bound:     81.133
Chi Sq stat:    108.236
Upper bound:    138.651

Centering errors: NONE

Estimated parameters: Cartesian (meters)
Name         X               Y             Z
DET1    568024.7216   -4690674.6437   4270188.8165
MIL1    172135.9981   -4668696.6438   4327808.3373
NLIB   -130934.5056   -4762291.7295   4226854.6497
SAG1    496374.9607   -4597431.5173   4378421.3469
STB1    212435.6805   -4528758.9117   4471353.7562
WLCI    248645.8004   -4828261.3139   4146460.1013

Estimated parameters: geodetic (ddd.mmsssssss)
Name    latitude       longitude      height
DET1    42.175045419  -83.054306691   145.0456
MIL1    43.000913079  -87.531840898   147.3691
NLIB    41.461772743  -91.342961873   207.0274
SAG1    43.374311944  -83.501595889   149.2217
STB1    44.474374793  -87.185158774   148.8385
WLCI    40.483026939  -87.030715032   180.4252

Trace of estimated dispersion matrix: 0.000186
Estimated reference variance: 1.0022
```

```
Estimated standard errors (scaled by sqrt estimated reference variance)
Name    std(X)   std(Y)   std(Z)   std(n)   std(e)   std(up)
           m        m        m        m        m        m
DET1    0.0011   0.0041   0.0037   0.0007   0.0012   0.0055
MIL1    0.0008   0.0040   0.0037   0.0007   0.0008   0.0054
NLIB    0.0008   0.0029   0.0024   0.0010   0.0008   0.0036
SAG1    0.0010   0.0040   0.0037   0.0007   0.0011   0.0054
STB1    0.0009   0.0043   0.0041   0.0009   0.0009   0.0059
WLCI    0.0009   0.0049   0.0043   0.0010   0.0009   0.0064

Observation Estimates
Obs#       From-To
Obs#    dX/dY/dZ      Obs.      Adjusted    Obs.       Stu.  Trad. Std.
          Obs.       Error         Obs.    Std. Dev.   Res.  Red # Rel #
Vec01: NLIB -> STB1
  1   343370.1879    0.0018    343370.186  0.00151   0.499  0.85  0.88
  2   233532.8082   -0.0096    233532.818  0.00653  -0.586  0.96  0.91
  3   244499.1146    0.0082    244499.106  0.00592   0.561  0.84  0.90
Vec02: NLIB -> SAG1
  4   627309.4588   -0.0075    627309.466  0.00169  -1.631  0.91  0.94
  5   164860.1847   -0.0276    164860.212  0.00637  -1.674  0.81  0.92
  6   151566.7236    0.0264    151566.697  0.00564   1.684  1.01  0.93
Vec03: NLIB -> DET1
  7   698959.2192   -0.0080    698959.227  0.00179  -1.430  0.96  0.95
  8    71617.0870    0.0012     71617.086  0.00648   0.066  0.97  0.92
  9    43334.1768    0.0100     43334.167  0.00568   0.684  0.84  0.91
Vec04: WLCI -> STB1
 10   -36210.1220   -0.0021    -36210.120  0.00108  -1.273  0.70  0.74
 11   299502.4067    0.0045    299502.402  0.00580   0.361  0.92  0.81
 12   324893.6501   -0.0048    324893.655  0.00515  -0.456  0.70  0.80
Vec05: MIL1 -> STB1
 13    40299.6816   -0.0008     40299.682  0.00093  -0.489  0.79  0.81
 14   139937.7255   -0.0065    139937.732  0.00462  -0.605  0.69  0.82
 15   143545.4233    0.0045    143545.419  0.00449   0.410  1.01  0.83
Vec06: MIL1 -> DET1
 16   395888.7290    0.0054    395888.724  0.00115   1.448  0.91  0.87
 17   -21978.0055   -0.0056    -21978.000  0.00413  -0.426  0.98  0.88
 18   -57619.5150    0.0058    -57619.521  0.00369   0.521  0.82  0.87
Vec07: STB1 -> SAG1
 19   283939.2827    0.0025    283939.280  0.00111   0.659  0.92  0.92
 20   -68672.6080   -0.0025    -68672.606  0.00457  -0.176  0.93  0.89
 21   -92932.4050    0.0043    -92932.409  0.00444   0.328  0.86  0.89
Vec08: SAG1 -> MIL1
 22  -324238.9628   -0.0001   -324238.963  0.00105  -0.036  0.93  0.92
 23   -71265.1285   -0.0020    -71265.127  0.00416  -0.181  0.89  0.89
 24   -50613.0072    0.0024    -50613.010  0.00388   0.236  0.86  0.88
Vec09: SAG1 -> DET1
 25    71649.7642    0.0033     71649.761  0.00099   1.190  0.89  0.90
 26   -93243.1321   -0.0057    -93243.126  0.00418  -0.471  0.97  0.92
 27  -108232.5245    0.0059   -108232.530  0.00379   0.583  0.83  0.90
Vec10: WLCI -> NLIB
 28  -379580.3089   -0.0029   -379580.306  0.00156  -0.804  0.84  0.84
 29    65969.5692   -0.0152     65969.584  0.00695  -1.283  0.62  0.79
 30    80394.5595    0.0110     80394.548  0.00607   0.983  0.94  0.81
Vec11: WLCI -> MIL1
```

```
 31   -76509.7968    0.0056    -76509.802  0.00095   3.234  0.79  0.81*
 32   159564.6675   -0.0027    159564.670  0.00532  -0.283  0.71  0.85
 33   181348.2377    0.0016    181348.236  0.00462   0.187  0.93  0.86
Vec12: WLCI -> SAG1
 34   247729.1634    0.0031    247729.160  0.00115   1.142  0.88  0.90
 35   230829.8005    0.0038    230829.797  0.00550   0.369  0.98  0.86
 36   231961.2424   -0.0033    231961.246  0.00473  -0.379  0.67  0.84
Vec13: WLCI -> DET1
 37   319378.9265    0.0053    319378.921  0.00120   1.968  0.86  0.87
 38   137586.6704    0.0002    137586.670  0.00550   0.016  0.89  0.87
 39   123728.7143   -0.0010    123728.715  0.00463  -0.112  0.78  0.86
Vec14: NLIB -> STB1
 40   343370.1774   -0.0087    343370.186  0.00151  -2.401  0.84  0.88
 41   233532.7983   -0.0195    233532.818  0.00653  -1.117  0.86  0.91
 42   244499.1267    0.0203    244499.106  0.00592   1.218  0.94  0.92
Vec15: NLIB -> SAG1
 43   627309.4586   -0.0077    627309.466  0.00169  -1.368  0.95  0.95
 44   164860.2095   -0.0028    164860.212  0.00637  -0.141  0.97  0.95
 45   151566.6960   -0.0012    151566.697  0.00564  -0.071  0.88  0.93
Vec16: DET1 -> MIL1
 46  -395888.7275   -0.0039   -395888.724  0.00115  -0.825  0.94  0.94
 47    21977.9926   -0.0073     21978.000  0.00413  -0.453  0.95  0.94
 48    57619.5322    0.0114     57619.521  0.00369   0.799  0.93  0.94
Vec17: STB1 -> MIL1
 49   -40299.6831   -0.0007    -40299.682  0.00093  -0.234  0.91  0.92
 50  -139937.7257    0.0063   -139937.732  0.00462   0.491  0.95  0.91
 51  -143545.4248   -0.0060   -143545.419  0.00449  -0.500  0.85  0.91
Vec18: STB1 -> SAG1
 52   283939.2848    0.0046    283939.280  0.00111   1.682  0.83  0.85
 53   -68672.6097   -0.0042    -68672.606  0.00457  -0.350  0.82  0.88
 54   -92932.4075    0.0018    -92932.409  0.00444   0.148  0.96  0.90
Vec19: STB1 -> DET1
 55   355589.0432    0.0021    355589.041  0.00122   0.557  0.92  0.91
 56  -161915.7338   -0.0018   -161915.732  0.00476  -0.142  0.91  0.91
 57  -201164.9447   -0.0050   -201164.940  0.00451  -0.423  0.88  0.91
Vec20: WLCI -> NLIB
 58  -379580.3079   -0.0019   -379580.306  0.00156  -0.760  0.71  0.73
 59    65969.5812   -0.0032     65969.584  0.00695  -0.270  0.82  0.72
 60    80394.5528    0.0043     80394.548  0.00607   0.429  0.63  0.71
Vec21: WLCI -> MIL1
 61   -76509.8036   -0.0012    -76509.802  0.00095  -0.722  0.79  0.81
 62   159564.6650   -0.0052    159564.670  0.00532  -0.491  0.76  0.85
 63   181348.2391    0.0030    181348.236  0.00462   0.309  0.93  0.86
Vec22: WLCI -> STB1
 64   -36210.1200   -0.0001    -36210.120  0.00108  -0.032  0.90  0.91
 65   299502.4000   -0.0022    299502.402  0.00580  -0.194  0.81  0.84
 66   324893.6528   -0.0021    324893.655  0.00515  -0.201  0.86  0.84
Vec23: WLCI -> SAG1
 67   247729.1604    0.0001    247729.160  0.00115   0.035  0.88  0.88
 68   230829.7950   -0.0017    230829.797  0.00550  -0.160  0.72  0.85
 69   231961.2460    0.0003    231961.246  0.00473   0.035  0.94  0.86
Vec24: WLCI -> DET1
 70   319378.9210   -0.0002    319378.921  0.00120  -0.078  0.85  0.86
 71   137586.6583   -0.0119    137586.670  0.00550  -1.037  0.92  0.87
 72   123728.7187    0.0034    123728.715  0.00463   0.352  0.80  0.87
```

```
Vec25: SAG1 -> MIL1
 73  -324238.9627   -0.0000   -324238.963  0.00105  -0.009  0.93  0.93
 74   -71265.1298   -0.0033    -71265.127  0.00416  -0.251  0.92  0.91
 75   -50613.0076    0.0020    -50613.010  0.00388   0.162  0.90  0.91
Vec2 SAG1 -> DET1
 76    71649.7560   -0.0049     71649.761  0.00099  -1.770  0.88  0.88
 77   -93243.1265   -0.0001    -93243.126  0.00418  -0.005  0.88  0.91
 78  -108232.5321   -0.0017   -108232.530  0.00379  -0.140  0.95  0.92
Vec27: SAG1 -> MIL1
 79  -324238.9561    0.0066   -324238.963  0.00105   1.371  0.96  0.95
 80   -71265.1293   -0.0028    -71265.127  0.00416  -0.198  0.93  0.93
 81   -50613.0039    0.0057    -50613.010  0.00388   0.431  0.92  0.93
Vec28: SAG1 -> DET1
 82    71649.7569   -0.0040     71649.761  0.00099  -1.445  0.87  0.87
 83   -93243.1248    0.0016    -93243.126  0.00418   0.125  0.88  0.91
 84  -108232.5287    0.0017   -108232.530  0.00379   0.139  0.94  0.92
Vec29: WLCI -> MIL1
 85   -76509.8078   -0.0054    -76509.802  0.00095  -1.953  0.91  0.91
 86   159564.6742    0.0040    159564.670  0.00532   0.274  0.92  0.93
 87   181348.2370    0.0009    181348.236  0.00462   0.071  0.90  0.93
Vec30: WLCI -> STB1
 88   -36210.1189    0.0010    -36210.120  0.00108   0.269  0.94  0.94
 89   299502.4108    0.0086    299502.402  0.00580   0.550  0.92  0.92
 90   324893.6546   -0.0003    324893.655  0.00515  -0.023  0.89  0.92
Vec31: WLCI -> SAG1
 91   247729.1551   -0.0052    247729.160  0.00115  -1.388  0.93  0.93
 92   230829.8032    0.0065    230829.797  0.00550   0.415  0.92  0.94
 93   231961.2481    0.0024    231961.246  0.00473   0.175  0.93  0.94
Vec32: WLCI -> DET1
 94   319378.9159   -0.0053    319378.921  0.00120  -1.421  0.91  0.92
 95   137586.6688   -0.0014    137586.670  0.00550  -0.091  0.87  0.93
 96   123728.7271    0.0118    123728.715  0.00463   0.847  0.97  0.93
Vec33: DET1 -> MIL1
 97  -395888.7206    0.0030   -395888.724  0.00115   0.514  0.96  0.96
 98    21978.0014    0.0015     21978.000  0.00413   0.086  0.94  0.94
 99    57619.5230    0.0022     57619.521  0.00369   0.143  0.94  0.94
Vec34: NLIB -> MIL1
100   303070.5033   -0.0003    303070.504  0.00147  -0.136  0.75  0.83
101    93595.0934    0.0076     93595.086  0.00640   0.577  0.97  0.89
102   100953.6803   -0.0073    100953.688  0.00569  -0.641  0.75  0.87
Vec35: NLIB -> STB1
103   343370.1892    0.0031    343370.186  0.00151   0.858  0.90  0.91
104   233532.8166   -0.0012    233532.818  0.00653  -0.091  0.77  0.85
105   244499.1077    0.0013    244499.106  0.00592   0.101  0.92  0.85
Vec36: NLIB -> WLCI
106   379580.3079    0.0019    379580.306  0.00156   0.526  0.90  0.90
107   -65969.5853   -0.0009    -65969.584  0.00695  -0.064  0.79  0.83
108   -80394.5519   -0.0034    -80394.548  0.00607  -0.278  0.87  0.84
Vec37: NLIB -> SAG1
109   627309.4673    0.0010    627309.466  0.00169   0.279  0.84  0.88
110   164860.2071   -0.0052    164860.212  0.00637  -0.360  0.95  0.91
111   151566.6992    0.0020    151566.697  0.00564   0.161  0.85  0.90
Vec38: NLIB -> DET1
112   698959.2342    0.0070    698959.227  0.00179   1.530  0.95  0.92
113    71617.0964    0.0106     71617.086  0.00648   0.739  0.87  0.90
```

```
114    43334.1605   -0.0063     43334.167  0.00568  -0.503  0.90  0.90
Vec39: STB1 -> MIL1
115   -40299.6839   -0.0015    -40299.682  0.00093  -0.841  0.78  0.79
116  -139937.7382   -0.0062   -139937.732  0.00462  -0.476  0.82  0.90
117  -143545.4220   -0.0032   -143545.419  0.00449  -0.244  0.98  0.91
Vec40: STB1 -> SAG1
118   283939.2732   -0.0070    283939.280  0.00111  -1.866  0.93  0.92
119   -68672.6091   -0.0036    -68672.606  0.00457  -0.255  0.96  0.90
120   -92932.4100   -0.0007    -92932.409  0.00444  -0.057  0.84  0.90
Vec41: STB1 -> DET1
121   355589.0380   -0.0031    355589.041  0.00122  -0.837  0.90  0.90
122  -161915.7331   -0.0011   -161915.732  0.00476  -0.082  0.92  0.91
123  -201164.9407   -0.0010   -201164.940  0.00451  -0.080  0.89  0.91

Sum of traditional redundancy numbers   =  108.00
Sum of standardized reliability numbers =  109.03

Estimated baseline outliers and minimum detectible outliers in meters
alpha = 0.01, beta = 0.80, r1 = 3, r2 = 105, non-central param. = 8.08
F(0.01;3,105) = 3.97
No. from  to  est. outlier[dX,dY,dZ]  T   min. detect.[dX,dY,dZ] Ex Rel
 1 NLIB->STB1 [ 0.006, 0.003,-0.003] 1.35 [0.0077,-0.0058,0.0161] 0.643
 2 NLIB->SAG1 [-0.005,-0.022, 0.019] 1.94 [0.0081,-0.0059,0.0164] 0.513
 3 NLIB->DET1 [-0.004, 0.017,-0.003] 2.26 [0.0075,-0.0056,0.0151] 0.627
 4 WLCI->STB1 [-0.001, 0.008,-0.009] 0.36 [0.0046,-0.0032,0.0092] 2.893
 5 MIL1->STB1 [-0.001,-0.012, 0.009] 1.03 [0.0039,-0.0027,0.0080] 1.852
 6 MIL1->DET1 [ 0.007, 0.000, 0.002] 1.82 [0.0052,-0.0035,0.0103] 1.172
 7 STB1->SAG1 [ 0.003,-0.003, 0.005] 0.25 [0.0069,-0.0043,0.0139] 0.985
 8 SAG1->MIL1 [-0.001, 0.000, 0.000] 0.03 [0.0059,-0.0039,0.0118] 0.917
 9 SAG1->DET1 [ 0.004,-0.006, 0.006] 0.50 [0.0056,-0.0034,0.0108] 0.883
10 WLCI->NLIB [-0.006,-0.020, 0.014] 1.35 [0.0068,-0.0056,0.0140] 2.032
11 WLCI->MIL1 [ 0.006,-0.005, 0.004] 3.27 [0.0043,-0.0032,0.0086] 1.730
12 WLCI->SAG1 [-0.000, 0.006,-0.005] 0.12 [0.0052,-0.0036,0.0102] 1.373
13 WLCI->DET1 [ 0.003, 0.001,-0.002] 0.67 [0.0050,-0.0035,0.0098] 1.150
14 NLIB->STB1 [-0.007,-0.016, 0.021] 2.51 [0.0079,-0.0060,0.0165] 0.836
15 NLIB->SAG1 [-0.001, 0.015,-0.017] 0.44 [0.0094,-0.0068,0.0190] 0.485
16 DET1->MIL1 [-0.004,-0.008, 0.012] 0.68 [0.0075,-0.0050,0.0147] 0.613
17 STB1->MIL1 [-0.001, 0.009,-0.004] 0.52 [0.0058,-0.0040,0.0118] 0.776
18 STB1->SAG1 [ 0.004,-0.007, 0.007] 0.88 [0.0050,-0.0031,0.0100] 1.288
19 STB1->DET1 [-0.001,-0.004,-0.002] 0.70 [0.0055,-0.0035,0.0109] 0.845
20 WLCI->NLIB [-0.003,-0.000, 0.006] 1.16 [0.0060,-0.0049,0.0122] 3.127
21 WLCI->MIL1 [-0.001,-0.002, 0.002] 0.19 [0.0042,-0.0031,0.0084] 1.706
22 WLCI->STB1 [ 0.001, 0.002,-0.005] 0.25 [0.0059,-0.0042,0.0118] 1.079
23 WLCI->SAG1 [ 0.001, 0.004,-0.002] 0.26 [0.0054,-0.0037,0.0107] 0.987
24 WLCI->DET1 [ 0.000,-0.011, 0.003] 1.89 [0.0049,-0.0034,0.0095] 1.194
25 SAG1->MIL1 [ 0.001,-0.003, 0.002] 0.04 [0.0064,-0.0042,0.0127] 0.719
26 SAG1->DET1 [-0.006, 0.001,-0.003] 1.63 [0.0055,-0.0034,0.0106] 0.912
27 SAG1->MIL1 [ 0.006,-0.002, 0.004] 0.64 [0.0074,-0.0049,0.0147] 0.529
28 SAG1->DET1 [-0.005, 0.002,-0.002] 1.04 [0.0054,-0.0033,0.0104] 1.027
29 WLCI->MIL1 [-0.004, 0.001,-0.002] 0.85 [0.0059,-0.0044,0.0118] 0.702
30 WLCI->STB1 [ 0.005, 0.007,-0.004] 1.04 [0.0072,-0.0051,0.0146] 0.605
31 WLCI->SAG1 [-0.003, 0.004,-0.001] 0.39 [0.0071,-0.0048,0.0139] 0.520
32 WLCI->DET1 [-0.004,-0.006, 0.012] 2.13 [0.0070,-0.0049,0.0136] 0.488
33 DET1->MIL1 [ 0.003, 0.002, 0.002] 0.29 [0.0086,-0.0058,0.0170] 0.460
34 NLIB->MIL1 [-0.002, 0.007,-0.007] 0.48 [0.0049,-0.0039,0.0103] 1.206
```

```
35 NLIB->STB1 [ 0.002,-0.007, 0.005] 0.23 [0.0067,-0.0051,0.0139] 0.764
36 NLIB->WLCI [ 0.000,-0.004,-0.001] 0.26 [0.0061,-0.0050,0.0124] 1.248
37 NLIB->SAG1 [-0.002,-0.013, 0.008] 0.94 [0.0060,-0.0043,0.0121] 0.848
38 NLIB->DET1 [ 0.006, 0.010,-0.004] 2.49 [0.0060,-0.0045,0.0121] 0.697
39 STB1->MIL1 [-0.000,-0.005,-0.004] 1.80 [0.0049,-0.0034,0.0100] 1.617
40 STB1->SAG1 [-0.006,-0.001, 0.001] 1.75 [0.0056,-0.0035,0.0112] 0.872
41 STB1->DET1 [ 0.000, 0.003,-0.001] 0.09 [0.0054,-0.0034,0.0106] 0.851
```

APPENDIX F

SCLESS for CORS Validation, 41 Observed Baseline Vectors

```
GPS observation variances and covariances scaled by 96.000 beginning at
observation 1.
The 3x3 block diagonal covariance matrix is replaced by the full
(session) matrix.
Adjustment type: Stochastically Constrained Least-Squares Solution
Units: dms, meters
No of observations    :  123
No. parameters        : - 18
Rank of K             : + 18
                        ----
System redundancy     :  123

Adjustment PASSED the Chi Square test at the 95% Confidence Level
Lower bound:     94.195
Chi Sq stat:    118.150
Upper bound:    155.589

Centering errors: NONE

Estimated parameters: Cartesian (meters)
Name          X              Y              Z
DET1     568024.7218  -4690674.6439   4270188.8167
MIL1     172135.9983  -4668696.6449   4327808.3384
NLIB    -130934.5057  -4762291.7292   4226854.6496
SAG1     496374.9608  -4597431.5176   4378421.3473
STB1     212435.6806  -4528758.9115   4471353.7560
WLCI     248645.8007  -4828261.3142   4146460.1010

Estimated parameters: geodetic (ddd.mmsssssss)
Name    latitude        longitude       height
DET1    42.1750454190  -83.0543066907  145.0459
MIL1    43.0009130796  -87.5318408978  147.3706
NLIB    41.4617727438  -91.3429618743  207.0271
SAG1    43.3743119448  -83.5015958896  149.2222
STB1    44.4743747940  -87.1851587739  148.8382
WLCI    40.4830269382  -87.0307150316  180.4252

Trace of estimated dispersion matrix: 0.000200
Estimated reference variance:       0.9606
```

```
Estimated standard errors (scaled by sqrt estimated reference variance)
Name    std(X)   std(Y)   std(Z)   std(n)   std(e)   std(up)
           m        m        m        m        m        m
DET1    0.0019   0.0041   0.0037   0.0021   0.0019   0.0051
MIL1    0.0018   0.0040   0.0038   0.0020   0.0018   0.0051
NLIB    0.0017   0.0032   0.0029   0.0021   0.0017   0.0037
SAG1    0.0018   0.0040   0.0038   0.0021   0.0018   0.0051
STB1    0.0018   0.0042   0.0040   0.0021   0.0018   0.0054
WLCI    0.0018   0.0046   0.0041   0.0022   0.0018   0.0058

Observation Estimates
Obs#        From-To
Obs#    dX/dY/dZ      Obs.      Adjusted    Obs.       Stu.  Trad. Std.
          Obs.       Error        Obs.    Std. Dev.    Res.  Red # Rel #
Vec01: NLIB -> STB1
  1   343370.1879    0.0016    343370.186  0.00136   0.438  0.87  0.89
  2   233532.8082   -0.0095    233532.818  0.00452  -0.571  0.95  0.92
  3   244499.1146    0.0082    244499.106  0.00421   0.558  0.89  0.91
Vec02: NLIB -> SAG1
  4   627309.4588   -0.0078    627309.467  0.00151  -1.712  0.92  0.95
  5   164860.1847   -0.0269    164860.212  0.00436  -1.606  0.88  0.93
  6   151566.7236    0.0259    151566.698  0.00392   1.633  0.98  0.94
Vec03: NLIB -> DET1
  7   698959.2192   -0.0083    698959.228  0.00160  -1.505  0.96  0.96
  8    71617.0870    0.0018     71617.085  0.00444   0.102  0.96  0.93
  9    43334.1768    0.0097     43334.167  0.00390   0.653  0.89  0.92
Vec04: WLCI -> STB1
 10   -36210.1220   -0.0019    -36210.120  0.00104  -1.150  0.71  0.74
 11   299502.4067    0.0040    299502.403  0.00500   0.318  0.93  0.82
 12   324893.6501   -0.0049    324893.655  0.00447  -0.462  0.74  0.81
Vec05: MIL1 -> STB1
 13    40299.6816   -0.0007     40299.682  0.00090  -0.384  0.79  0.81
 14   139937.7255   -0.0080    139937.734  0.00413  -0.746  0.73  0.83
 15   143545.4233    0.0058    143545.417  0.00401   0.537  0.99  0.84
Vec06: MIL1 -> DET1
 16   395888.7290    0.0055    395888.723  0.00110   1.507  0.92  0.87
 17   -21978.0055   -0.0065    -21977.999  0.00377  -0.504  0.98  0.88
 18   -57619.5150    0.0068    -57619.522  0.00338   0.615  0.84  0.87
Vec07: STB1 -> SAG1
 19   283939.2827    0.0024    283939.280  0.00107   0.657  0.92  0.92
 20   -68672.6080   -0.0018    -68672.606  0.00410  -0.134  0.93  0.90
 21   -92932.4050    0.0036    -92932.409  0.00398   0.284  0.88  0.89
Vec08: SAG1 -> MIL1
 22  -324238.9628   -0.0003   -324238.963  0.00101  -0.073  0.93  0.93
 23   -71265.1285   -0.0011    -71265.127  0.00379  -0.104  0.90  0.89
 24   -50613.0072    0.0017    -50613.009  0.00353   0.166  0.87  0.88
Vec09: SAG1 -> DET1
 25    71649.7642    0.0033     71649.761  0.00095   1.204  0.89  0.90
 26   -93243.1321   -0.0057    -93243.126  0.00381  -0.482  0.97  0.92
 27  -108232.5245    0.0061   -108232.531  0.00346   0.614  0.85  0.90
Vec10: WLCI -> NLIB
 28  -379580.3089   -0.0024   -379580.306  0.00140  -0.680  0.86  0.86
 29    65969.5692   -0.0158     65969.585  0.00490  -1.263  0.78  0.81
 30    80394.5595    0.0109     80394.549  0.00426   0.925  0.93  0.82
Vec11: WLCI -> MIL1
```

```
 31   -76509.7968    0.0056    -76509.802  0.00091   3.310  0.79  0.82*
 32   159564.6675   -0.0017    159564.669  0.00464  -0.180  0.75  0.86
 33   181348.2377    0.0002    181348.238  0.00406   0.022  0.92  0.87
Vec12: WLCI -> SAG1
 34   247729.1634    0.0033    247729.160  0.00110   1.226  0.88  0.90
 35   230829.8005    0.0039    230829.797  0.00478   0.374  0.96  0.86
 36   231961.2424   -0.0040    231961.246  0.00414  -0.457  0.72  0.84
Vec13: WLCI -> DET1
 37   319378.9265    0.0054    319378.921  0.00114   2.057  0.86  0.87
 38   137586.6704    0.0002    137586.670  0.00478   0.018  0.90  0.88
 39   123728.7143   -0.0014    123728.716  0.00405  -0.164  0.81  0.87
Vec14: NLIB -> STB1
 40   343370.1774   -0.0089    343370.186  0.00136  -2.486  0.86  0.89
 41   233532.7983   -0.0194    233532.818  0.00452  -1.099  0.91  0.92
 42   244499.1267    0.0203    244499.106  0.00421   1.213  0.94  0.92
Vec15: NLIB -> SAG1
 43   627309.4586   -0.0080    627309.467  0.00151  -1.439  0.96  0.95
 44   164860.2095   -0.0021    164860.212  0.00436  -0.105  0.97  0.95
 45   151566.6960   -0.0017    151566.698  0.00392  -0.102  0.92  0.94
Vec16: DET1 -> MIL1
 46  -395888.7275   -0.0040   -395888.723  0.00110  -0.866  0.95  0.94
 47    21977.9926   -0.0064     21977.999  0.00377  -0.403  0.95  0.94
 48    57619.5322    0.0104     57619.522  0.00338   0.745  0.94  0.94
Vec17: STB1 -> MIL1
 49   -40299.6831   -0.0008    -40299.682  0.00090  -0.310  0.91  0.92
 50  -139937.7257    0.0078   -139937.734  0.00413   0.612  0.95  0.92
 51  -143545.4248   -0.0073   -143545.417  0.00401  -0.618  0.87  0.91
Vec18: STB1 -> SAG1
 52   283939.2848    0.0045    283939.280  0.00107   1.691  0.83  0.85
 53   -68672.6097   -0.0035    -68672.606  0.00410  -0.301  0.84  0.89
 54   -92932.4075    0.0011    -92932.409  0.00398   0.097  0.95  0.90
Vec19: STB1 -> DET1
 55   355589.0432    0.0020    355589.041  0.00116   0.546  0.93  0.91
 56  -161915.7338   -0.0013   -161915.733  0.00424  -0.100  0.92  0.91
 57  -201164.9447   -0.0054   -201164.939  0.00402  -0.459  0.89  0.91
Vec20: WLCI -> NLIB
 58  -379580.3079   -0.0014   -379580.306  0.00140  -0.569  0.76  0.76
 59    65969.5812   -0.0038     65969.585  0.00490  -0.305  0.87  0.74
 60    80394.5528    0.0042     80394.549  0.00426   0.388  0.75  0.74
Vec21: WLCI -> MIL1
 61   -76509.8036   -0.0012    -76509.802  0.00091  -0.714  0.80  0.81
 62   159564.6650   -0.0042    159564.669  0.00464  -0.399  0.79  0.86
 63   181348.2391    0.0016    181348.238  0.00406   0.163  0.93  0.87
Vec22: WLCI -> STB1
 64   -36210.1200    0.0001    -36210.120  0.00104   0.053  0.90  0.91
 65   299502.4000   -0.0027    299502.403  0.00500  -0.239  0.84  0.84
 66   324893.6528   -0.0022    324893.655  0.00447  -0.207  0.87  0.84
Vec23: WLCI -> SAG1
 67   247729.1604    0.0003    247729.160  0.00110   0.100  0.88  0.89
 68   230829.7950   -0.0016    230829.797  0.00478  -0.151  0.76  0.86
 69   231961.2460   -0.0004    231961.246  0.00414  -0.037  0.93  0.86
Vec24: WLCI -> DET1
 70   319378.9210   -0.0001    319378.921  0.00114  -0.022  0.85  0.87
 71   137586.6583   -0.0119    137586.670  0.00478  -1.033  0.92  0.87
 72   123728.7187    0.0030    123728.716  0.00405   0.304  0.83  0.87
```

```
Vec25: SAG1 -> MIL1
 73  -324238.9627   -0.0002   -324238.963  0.00101  -0.046  0.93  0.93
 74   -71265.1298   -0.0024    -71265.127  0.00379  -0.188  0.92  0.91
 75   -50613.0076    0.0013    -50613.009  0.00353   0.105  0.91  0.91
Vec26: SAG1 -> DET1
 76    71649.7560   -0.0049     71649.761  0.00095  -1.810  0.88  0.88
 77   -93243.1265   -0.0001    -93243.126  0.00381  -0.010  0.90  0.91
 78  -108232.5321   -0.0015   -108232.531  0.00346  -0.123  0.95  0.92
Vec27: SAG1 -> MIL1
 79  -324238.9561    0.0064   -324238.963  0.00101   1.369  0.96  0.95
 80   -71265.1293   -0.0019    -71265.127  0.00379  -0.139  0.94  0.93
 81   -50613.0039    0.0050    -50613.009  0.00353   0.382  0.92  0.93
Vec28: SAG1 -> DET1
 82    71649.7569   -0.0040     71649.761  0.00095  -1.479  0.88  0.87
 83   -93243.1248    0.0016    -93243.126  0.00381   0.122  0.89  0.91
 84  -108232.5287    0.0019   -108232.531  0.00346   0.161  0.94  0.92
Vec29: WLCI -> MIL1
 85   -76509.8078   -0.0054    -76509.802  0.00091  -1.978  0.91  0.92
 86   159564.6742    0.0050    159564.669  0.00464   0.341  0.93  0.93
 87   181348.2370   -0.0005    181348.238  0.00406  -0.039  0.91  0.93
Vec30: WLCI -> STB1
 88   -36210.1189    0.0012    -36210.120  0.00104   0.336  0.94  0.94
 89   299502.4108    0.0081    299502.403  0.00500   0.519  0.93  0.92
 90   324893.6546   -0.0004    324893.655  0.00447  -0.029  0.90  0.92
Vec31: WLCI -> SAG1
 91   247729.1551   -0.0050    247729.160  0.00110  -1.368  0.93  0.93
 92   230829.8032    0.0066    230829.797  0.00478   0.424  0.93  0.94
 93   231961.2481    0.0017    231961.246  0.00414   0.126  0.93  0.94
Vec32: WLCI -> DET1
 94   319378.9159   -0.0052    319378.921  0.00114  -1.406  0.91  0.92
 95   137586.6688   -0.0014    137586.670  0.00478  -0.090  0.89  0.93
 96   123728.7271    0.0114    123728.716  0.00405   0.822  0.96  0.93
Vec33: DET1 -> MIL1
 97  -395888.7206    0.0029   -395888.723  0.00110   0.504  0.96  0.96
 98    21978.0014    0.0024     21977.999  0.00377   0.142  0.95  0.94
 99    57619.5230    0.0012     57619.522  0.00338   0.082  0.94  0.94
Vec34: NLIB -> MIL1
100   303070.5033   -0.0008    303070.504  0.00132  -0.303  0.77  0.84
101    93595.0934    0.0092     93595.084  0.00436   0.671  0.94  0.89
102   100953.6803   -0.0086    100953.689  0.00393  -0.723  0.83  0.88
Vec35: NLIB -> STB1
103   343370.1892    0.0029    343370.186  0.00136   0.800  0.91  0.92
104   233532.8166   -0.0011    233532.818  0.00452  -0.082  0.84  0.86
105   244499.1077    0.0013    244499.106  0.00421   0.105  0.91  0.86
Vec36: NLIB -> WLCI
106   379580.3079    0.0014    379580.306  0.00140   0.400  0.90  0.91
107   -65969.5853   -0.0003    -65969.585  0.00490  -0.020  0.85  0.85
108   -80394.5519   -0.0033    -80394.549  0.00426  -0.255  0.88  0.85
Vec37: NLIB -> SAG1
109   627309.4673    0.0007    627309.467  0.00151   0.197  0.86  0.88
110   164860.2071   -0.0045    164860.212  0.00436  -0.303  0.93  0.91
111   151566.6992    0.0015    151566.698  0.00392   0.115  0.89  0.91
Vec38: NLIB -> DET1
112   698959.2342    0.0067    698959.228  0.00160   1.474  0.95  0.93
113    71617.0964    0.0112     71617.085  0.00444   0.762  0.90  0.91
```

```
114    43334.1605   -0.0066     43334.167  0.00390  -0.512  0.91  0.90
Vec39: STB1 -> MIL1
115   -40299.6839   -0.0016    -40299.682  0.00090  -0.971  0.78  0.79
116  -139937.7382   -0.0047   -139937.734  0.00413  -0.365  0.85  0.91
117  -143545.4220   -0.0045   -143545.417  0.00401  -0.351  0.98  0.91
Vec40: STB1 -> SAG1
118   283939.2732   -0.0071    283939.280  0.00107  -1.917  0.93  0.92
119   -68672.6091   -0.0029    -68672.606  0.00410  -0.213  0.96  0.91
120   -92932.4100   -0.0014    -92932.409  0.00398  -0.106  0.86  0.90
Vec41: STB1 -> DET1
121   355589.0380   -0.0032    355589.041  0.00116  -0.874  0.90  0.90
122  -161915.7331   -0.0006   -161915.733  0.00424  -0.042  0.93  0.91
123  -201164.9407   -0.0014   -201164.939  0.00402  -0.111  0.90  0.91


Sum of traditional redundancy numbers   =  123.00
Sum of standardized reliability numbers =  123.22

Estimated baseline outliers and minimum detectable outliers in meters
alpha = 0.01, beta = 0.80, r1 = 3, r2 = 120, non-central param. = 8.08
F(0.01;3,120) = 3.95
No. from  to  est. outlier[dX,dY,dZ]  T   min. detect.[dX,dY,dZ] Ex Rel
 1 NLIB->STB1 [ 0.006, 0.002,-0.003] 1.32 [0.0077,-0.0058,0.0160] 0.589
 2 NLIB->SAG1 [-0.005,-0.021, 0.018] 1.98 [0.0081,-0.0059,0.0163] 0.477
 3 NLIB->DET1 [-0.004, 0.017,-0.004] 2.43 [0.0075,-0.0055,0.0150] 0.558
 4 WLCI->STB1 [-0.001, 0.008,-0.009] 0.43 [0.0045,-0.0032,0.0091] 2.746
 5 MIL1->STB1 [-0.001,-0.012, 0.010] 1.15 [0.0039,-0.0027,0.0080] 1.794
 6 MIL1->DET1 [ 0.007, 0.000, 0.002] 2.07 [0.0052,-0.0035,0.0103] 1.131
 7 STB1->SAG1 [ 0.003,-0.002, 0.004] 0.25 [0.0069,-0.0043,0.0139] 0.947
 8 SAG1->MIL1 [-0.001, 0.001,-0.001] 0.04 [0.0059,-0.0039,0.0118] 0.888
 9 SAG1->DET1 [ 0.004,-0.006, 0.006] 0.53 [0.0056,-0.0034,0.0108] 0.846
10 WLCI->NLIB [-0.005,-0.018, 0.012] 1.33 [0.0067,-0.0055,0.0138] 1.737
11 WLCI->MIL1 [ 0.006,-0.003, 0.002] 3.24 [0.0043,-0.0032,0.0086] 1.670
12 WLCI->SAG1 [-0.000, 0.006,-0.005] 0.14 [0.0052,-0.0036,0.0102] 1.296
13 WLCI->DET1 [ 0.004, 0.001,-0.002] 0.79 [0.0050,-0.0035,0.0098] 1.102
14 NLIB->STB1 [-0.007,-0.015, 0.021] 2.72 [0.0079,-0.0059,0.0164] 0.757
15 NLIB->SAG1 [-0.001, 0.015,-0.017] 0.48 [0.0093,-0.0068,0.0189] 0.433
16 DET1->MIL1 [-0.004,-0.007, 0.011] 0.67 [0.0074,-0.0050,0.0147] 0.595
17 STB1->MIL1 [-0.001, 0.010,-0.005] 0.59 [0.0058,-0.0039,0.0118] 0.749
18 STB1->SAG1 [ 0.004,-0.007, 0.006] 0.93 [0.0050,-0.0031,0.0100] 1.256
19 STB1->DET1 [-0.001,-0.004,-0.002] 0.69 [0.0055,-0.0035,0.0108] 0.820
20 WLCI->NLIB [-0.002,-0.000, 0.005] 0.93 [0.0058,-0.0048,0.0119] 2.594
21 WLCI->MIL1 [-0.001,-0.001, 0.001] 0.23 [0.0042,-0.0031,0.0084] 1.644
22 WLCI->STB1 [ 0.001, 0.001,-0.004] 0.26 [0.0059,-0.0042,0.0118] 1.013
23 WLCI->SAG1 [ 0.001, 0.004,-0.003] 0.29 [0.0054,-0.0037,0.0107] 0.958
24 WLCI->DET1 [ 0.000,-0.011, 0.002] 1.94 [0.0049,-0.0034,0.0095] 1.150
25 SAG1->MIL1 [ 0.001,-0.002, 0.001] 0.03 [0.0063,-0.0042,0.0126] 0.698
26 SAG1->DET1 [-0.006, 0.001,-0.002] 1.69 [0.0055,-0.0034,0.0106] 0.895
27 SAG1->MIL1 [ 0.006,-0.001, 0.003] 0.63 [0.0074,-0.0049,0.0147] 0.515
28 SAG1->DET1 [-0.005, 0.002,-0.001] 1.10 [0.0053,-0.0033,0.0104] 1.008
29 WLCI->MIL1 [-0.004, 0.002,-0.003] 0.93 [0.0059,-0.0044,0.0118] 0.684
30 WLCI->STB1 [ 0.005, 0.006,-0.003] 1.09 [0.0072,-0.0051,0.0146] 0.579
31 WLCI->SAG1 [-0.003, 0.004,-0.001] 0.35 [0.0071,-0.0048,0.0139] 0.506
32 WLCI->DET1 [-0.004,-0.006, 0.012] 2.12 [0.0070,-0.0049,0.0136] 0.475
33 DET1->MIL1 [ 0.003, 0.003, 0.001] 0.29 [0.0086,-0.0058,0.0170] 0.449
```

```
34 NLIB->MIL1 [-0.003, 0.009,-0.008] 0.64 [0.0049,-0.0039,0.0102] 1.122
35 NLIB->STB1 [ 0.002,-0.007, 0.006] 0.25 [0.0067,-0.0050,0.0139] 0.729
36 NLIB->WLCI [ 0.000,-0.004,-0.000] 0.19 [0.0060,-0.0050,0.0124] 1.172
37 NLIB->SAG1 [-0.002,-0.012, 0.007] 0.94 [0.0059,-0.0043,0.0121] 0.791
38 NLIB->DET1 [ 0.006, 0.011,-0.004] 2.67 [0.0060,-0.0045,0.0121] 0.666
39 STB1->MIL1 [-0.001,-0.004,-0.005] 1.85 [0.0049,-0.0033,0.0100] 1.582
40 STB1->SAG1 [-0.006,-0.001, 0.000] 1.78 [0.0056,-0.0035,0.0112] 0.847
41 STB1->DET1 [ 0.000, 0.003,-0.001] 0.10 [0.0054,-0.0034,0.0106] 0.829
```

APPENDIX G

RLESS for New Fiducial Points, 23 Observed Baseline Vectors

```
The 3x3 block diagonal covariance matrix is replaced by a full
(session) matrix

Adjustment type: Restricted Least-Squares Solution
Ellipsoid: WGS84
Units: dms, meters

No of observations     :   69
Rank of A              : - 24
                         ----
System redundancy      :   45

Adjustment FAILED the Chi Square test at the 95% Confidence Level
Lower bound:     28.366
Chi Sq stat:    578.584
Upper bound:     65.410

Centering errors:
Name   horiz[m] vert [m]
BEHD   0.003     0.000
G317   0.003     0.000
MBYC   0.003     0.000

Estimated parameters: Cartesian (meters)
Name          X              Y             Z
DET1     568024.7380  -4690674.6058   4270188.7941
MIL1     172135.9921  -4668696.5884   4327808.3152
NLIB    -130934.5086  -4762291.7268   4226854.6508
SAG1     496374.9671  -4597431.4948   4378421.3478
STB1     212435.6860  -4528758.8706   4471353.7460
WLCI     248645.8175  -4828261.2670   4146460.0581
BEHD     295059.6979  -4728575.1879   4256061.8012
G317     307138.8258  -4649646.6527   4340747.2254
MBYC     310880.0646  -4679085.7523   4308925.6514

Estimated parameters: geodetic (ddd.mmsssssss)
Name       latitude       longitude       height
DET1    42.1750454433    -83.0543066003     145.0041
MIL1    43.0009131499    -87.5318409158     147.3134
NLIB    41.4617727516    -91.3429618869     207.0262
SAG1    43.3743119950    -83.5015958511     149.2066
STB1    44.4743748635    -87.1851587407     148.8023
```

```
WLCI    40.4830269308    -87.0307149496     180.3622
BEHD    42.0731983055    -86.2545890069     156.0511
G317    43.0942931211    -86.1314659342     155.7018
MBYC    42.4614129938    -86.1155807242     143.2145

Trace of estimated dispersion matrix: 0.003497
Estimated reference variance: 12.8574

Estimated standard errors (scaled by sqrt estimated reference variance)
Name      std(X)   std(Y)   std(Z)    std(n)   std(e)   std(up)
            m        m        m         m        m        m
DET1    0.0141   0.0123   0.0119   0.0125   0.0143   0.0115
MIL1    0.0094   0.0101   0.0095   0.0091   0.0094   0.0104
NLIB    0.0000   0.0000   0.0000   0.0001   0.0000   0.0000
SAG1    0.0164   0.0136   0.0134   0.0154   0.0165   0.0111
STB1    0.0156   0.0134   0.0134   0.0157   0.0157   0.0105
WLCI    0.0106   0.0123   0.0116   0.0102   0.0106   0.0135
BEHD    0.0076   0.0090   0.0082   0.0064   0.0076   0.0103
G317    0.0146   0.0125   0.0124   0.0141   0.0146   0.0105
MBYC    0.0116   0.0110   0.0106   0.0109   0.0117   0.0105

Observation Estimates
Obs#       From-To
Obs#    dX/dY/dZ      Obs.      Adjusted    Obs.       Stu.  Trad. Std.
          Obs.       Error        Obs.    Std. Dev.    Res.  Red # Rel #
Vec01: MBYC -> G317
  1    -3741.2376    0.0012     -3741.239  0.00910    0.092  0.67  0.67
Vec01: MBYC -> G317
  1    -3741.2376    0.0012     -3741.239  0.00883    0.096  0.67  0.66
  2    29439.0952   -0.0045     29439.100  0.00820   -0.428  0.59  0.55
  3    31821.5696   -0.0044     31821.574  0.00821   -0.413  0.60  0.55
Vec02: SAG1 -> G317
  4  -189236.1424   -0.0011   -189236.141  0.00657   -0.127  0.64  0.64
  5   -52215.1555    0.0024    -52215.158  0.00722    0.286  0.57  0.55
  6   -37674.1125    0.0099    -37674.122  0.00701    1.140  0.60  0.58
Vec03: DET1 -> MBYC
  7  -257144.6615    0.0119   -257144.673  0.00678    1.399  0.61  0.62
  8    11588.8525   -0.0010     11588.853  0.00761   -0.110  0.56  0.55
  9    38736.8600    0.0027     38736.857  0.00730    0.311  0.59  0.58
Vec04: BEHD -> MBYC
 10    15820.3572   -0.0095     15820.367  0.00883   -0.762  0.67  0.66
 11    49489.4463    0.0108     49489.436  0.00837    0.997  0.61  0.54
 12    52863.8501   -0.0002     52863.850  0.00836   -0.015  0.62  0.55
Vec05: NLIB -> BEHD
 13   425994.2056   -0.0009    425994.206  0.00756   -0.109  0.53  0.53
 14    33716.5182   -0.0207     33716.539  0.00900   -2.040  0.57  0.58
 15    29207.1549    0.0045     29207.150  0.00822    0.462  0.61  0.61
Vec06: MIL1 -> BEHD
 16   122923.6979   -0.0078    122923.706  0.00640   -0.894  0.65  0.65
 17   -59878.6067   -0.0072    -59878.599  0.00725   -0.839  0.57  0.56
 18   -71746.5038    0.0103    -71746.514  0.00695    1.212  0.60  0.58
Vec07: G317 -> STB1
 19   -94703.1397    0.0001    -94703.140  0.00635    0.013  0.66  0.66
 20   120887.7846    0.0025    120887.782  0.00707    0.298  0.60  0.58
 21   130606.5082   -0.0124    130606.521  0.00712   -1.446  0.59  0.58
```

```
Vec08: NLIB -> BEHD
 22   425994.1987   -0.0078    425994.206  0.00756  -0.564  0.73  0.70
 23    33716.5441    0.0052     33716.539  0.00900   0.341  0.72  0.69
 24    29207.1454   -0.0050     29207.150  0.00822  -0.367  0.71  0.69
Vec09: MIL1 -> BEHD
 25   122923.6872   -0.0185    122923.706  0.00640  -1.981  0.67  0.67
 26   -59878.5719    0.0276    -59878.599  0.00725   2.485  0.69  0.68
 27   -71746.5240   -0.0099    -71746.514  0.00695  -0.920  0.70  0.69
Vec10: MBYC -> BEHD
 28   -15820.3673   -0.0006    -15820.367  0.00883  -0.048  0.67  0.67
 29   -49489.4261    0.0094    -49489.436  0.00837   0.730  0.70  0.70
 30   -52863.8515   -0.0012    -52863.850  0.00836  -0.096  0.70  0.70
Vec11: G317 -> MBYC
 31     3741.2373   -0.0015      3741.239  0.00883  -0.120  0.67  0.67
 32   -29439.1032   -0.0035    -29439.100  0.00820  -0.276  0.71  0.72
 33   -31821.5683    0.0057    -31821.574  0.00821   0.444  0.71  0.72
Vec12: SAG1 -> G317
 34  -189236.1416   -0.0003   -189236.141  0.00657  -0.030  0.68  0.67
 35   -52215.1569    0.0010    -52215.158  0.00722   0.086  0.70  0.70
 36   -37674.1254   -0.0030    -37674.122  0.00701  -0.278  0.70  0.69
Vec13: DET1 -> MBYC
 37  -257144.6734    0.0000   -257144.673  0.00678   0.002  0.69  0.67
 38    11588.8359   -0.0176     11588.853  0.00761  -1.419  0.71  0.70
 39    38736.8742    0.0169     38736.857  0.00730   1.427  0.71  0.70
Vec14: STB1 -> G317
 40    94703.1393   -0.0005     94703.140  0.00635  -0.056  0.67  0.67
 41  -120887.7727    0.0094   -120887.782  0.00707   0.828  0.70  0.70
 42  -130606.5306   -0.0100   -130606.521  0.00712  -0.866  0.71  0.70
Vec15: BEHD -> WLCI
 43   -46413.8842   -0.0039    -46413.880  0.00781  -0.494  0.50  0.50
 44   -99686.0721    0.0070    -99686.079  0.01039   0.618  0.50  0.50
 45  -109601.7420    0.0011   -109601.743  0.00953   0.099  0.51  0.51
Vec16: NLIB -> BEHD
 46   425994.2335    0.0270    425994.206  0.00756   1.961  0.74  0.70
 47    33716.5709    0.0320     33716.539  0.00900   2.156  0.71  0.69
 48    29207.1412   -0.0092     29207.150  0.00822  -0.717  0.69  0.68
Vec17: MIL1 -> BEHD
 49   122923.7390    0.0333    122923.706  0.00640   3.507  0.68  0.67*
 50   -59878.6232   -0.0237    -59878.599  0.00725  -1.947  0.74  0.73
 51   -71746.5195   -0.0054    -71746.514  0.00695  -0.487  0.71  0.70
Vec18: MBYC -> BEHD
 52   -15820.3763   -0.0096    -15820.367  0.00883  -0.769  0.67  0.67
 53   -49489.4339    0.0016    -49489.436  0.00837   0.134  0.68  0.70
 54   -52863.8539   -0.0036    -52863.850  0.00836  -0.297  0.68  0.69
Vec19: G317 -> MBYC
 55     3741.2415    0.0027      3741.239  0.00883   0.216  0.67  0.67
 56   -29439.1022   -0.0025    -29439.100  0.00820  -0.208  0.69  0.71
 57   -31821.5851   -0.0111    -31821.574  0.00821  -0.927  0.68  0.70
Vec20: SAG1 -> G317
 58  -189236.1477   -0.0064   -189236.141  0.00657  -0.637  0.68  0.68
 59   -52215.1632   -0.0053    -52215.158  0.00722  -0.445  0.73  0.72
 60   -37674.1318   -0.0094    -37674.122  0.00701  -0.861  0.70  0.70
Vec21: DET1 -> MBYC
 61  -257144.6977   -0.0243   -257144.673  0.00678  -2.208  0.70  0.68
 62    11588.8736    0.0201     11588.853  0.00761   1.581  0.73  0.72
```

```
 63    38736.8391   -0.0182     38736.857  0.00730  -1.577  0.70  0.69
Vec22: STB1 -> G317
 64    94703.1446    0.0048     94703.140  0.00635   0.518  0.67  0.67
 65  -120887.7869   -0.0048   -120887.782  0.00707  -0.441  0.70  0.70
 66  -130606.5361   -0.0155   -130606.521  0.00712  -1.381  0.70  0.69
Vec23: BEHD -> WLCI
 67   -46413.8783    0.0020    -46413.880  0.00781   0.258  0.50  0.50
 68   -99686.1022   -0.0231    -99686.079  0.01039  -2.022  0.50  0.50
 69  -109601.7399    0.0032   -109601.743  0.00953   0.309  0.49  0.49

Sum of traditional redundancy numbers   =   45.00
Sum of standardized reliability numbers =   44.43
```

Appendix H

BLIMPBE for New Fiducial Points, 22 Observed Baseline Vectors, with $\overline{S}$ formed per (23)

```
GPS observation variances and covariances scaled by 48.0 beginning at
observation 1.
The 3x3 block diagonal covariance matrix is replaced by the full
(session) matrix.

Adjustment type: Best LInear Minimum Bias Estimation with the first 3
points selected

No of observations                        :   66
Rank of A                                 : - 24
                                            ----
System redundancy                         :   42

Adjustment PASSED the Chi Square test at the 95% Confidence Level
Lower bound:     38.027
Chi Sq stat:     72.769
Upper bound:     79.752

Centering errors:
Name    horiz    vert
          m        m
G317    0.003    0.000
BEHD    0.003    0.000
MBYC    0.003    0.000

Estimated parameters: Cartesian (meters):
Name             X               Y                Z
BEHD    295059.6897   -4728575.2211   4256061.8187
G317    307138.8157   -4649646.6862   4340747.2395
MBYC    310880.0534   -4679085.7858   4308925.6667
DET1    568024.7169   -4690674.6455   4270188.8137
MIL1    172136.0032   -4668696.6486   4327808.3443
NLIB   -130934.5086   -4762291.7268   4226854.6508
SAG1    496374.9552   -4597431.5162   4378421.3510
STB1    212435.6760   -4528758.9095   4471353.7544
WLCI    248645.8056   -4828261.3182   4146460.0933

Estimated parameters: geodetic (ddd.mmsssssss):
Name        latitude       longitude        height
BEHD   42.0731982767  -86.2545890513     156.0871
G317   43.0942930816  -86.1314659882     155.7354
MBYC   42.4614129585  -86.1155807829     143.2488
```

```
DET1   42.1750454097  -83.0543067125     145.0446
MIL1   43.0009130849  -87.5318408768     147.3775
NLIB   41.4617727516  -91.3429618869     207.0262
SAG1   43.3743119579  -83.5015959139     149.2233
STB1   44.4743747953  -87.1851587942     148.8355
WLCI   40.4830269102  -87.0307150116     180.4234

Trace of estimated dispersion matrix: 0.000548
Estimated reference variance: 1.2766

Estimated standard errors (scaled by sqrt estimated reference variance)
Name    std(X)   std(Y)   std(Z)   std(n)  std(e)   std(up)
          m        m        m        m        m        m
BEHD    0.0023   0.0101   0.0088    0.0027   0.0022  0.0132
G317    0.0022   0.0096   0.0088    0.0027   0.0021  0.0127
MBYC    0.0024   0.0100   0.0091    0.0028   0.0023  0.0132
DET1    0.0000   0.0000   0.0000    0.0000   0.0000  0.0003
MIL1    0.0000   0.0000   0.0000    0.0000   0.0000  0.0001
NLIB    0.0000   0.0000   0.0000    0.0001   0.0000  0.0000
SAG1    0.0000   0.0000   0.0000    0.0000   0.0000  0.0003
STB1    0.0000   0.0000   0.0000    0.0000   0.0000  0.0002
WLCI    0.0000   0.0000   0.0000    0.0000   0.0000  0.0002

Observation Estimates
Obs#  From-To
Obs#     dX/dY/dZ     Obs.      Adjusted    Obs.    Stu.  Trad.   Std.
           Obs.      Error         Obs.    Std. Dev. Res.  Red #   Rel #
Vec01: MBYC -> G317
  1    -3741.2376    0.0001    -3741.238  0.00284   0.012 0.82    0.87
  2    29439.0952   -0.0043    29439.100  0.01222  -0.318 0.66    0.86
  3    31821.5696   -0.0031    31821.573  0.01120  -0.249 0.67    0.85
Vec02: SAG1 -> G317
  4  -189236.1424   -0.0029  -189236.139  0.00221  -0.629 0.85    0.88
  5   -52215.1555    0.0145   -52215.170  0.00955   0.940 0.82    0.89
  6   -37674.1125   -0.0010   -37674.112  0.00882  -0.064 0.89    0.89
Vec03: DET1 -> MBYC
  7  -257144.6615    0.0020  -257144.663  0.00236   0.433 0.80    0.81
  8    11588.8525   -0.0072    11588.860  0.01000  -0.427 0.84    0.87
  9    38736.8600    0.0070    38736.853  0.00910   0.443 0.87    0.87
Vec04: BEHD -> MBYC
 10    15820.3572   -0.0065    15820.364  0.00297  -1.327 0.79    0.83
 11    49489.4463    0.0109    49489.435  0.01284   0.723 0.72    0.87
 12    52863.8501    0.0021    52863.848  0.01148   0.157 0.69    0.86
Vec05: NLIB -> BEHD
 13   425994.2056    0.0073   425994.198  0.00231   1.372 0.87    0.90
 14    33716.5182    0.0125    33716.506  0.01012   0.544 0.90    0.91
 15    29207.1549   -0.0130    29207.168  0.00885  -0.645 0.90    0.91
Vec06: MIL1 -> BEHD
 16   122923.6979    0.0114   122923.687  0.00231   2.851 0.80    0.86*
 17   -59878.6067   -0.0342   -59878.573  0.01012  -2.137 0.87    0.88
 18   -71746.5038    0.0218   -71746.526  0.00885   1.546 0.81    0.87
Vec07: G317 -> STB1
 19   -94703.1397    0.0000   -94703.140  0.00221   0.011 0.81    0.85
 20   120887.7846    0.0079   120887.777  0.00955   0.511 0.86    0.90
 21   130606.5082   -0.0067   130606.515  0.00882  -0.450 0.87    0.90
```

```
Vec08: NLIB -> BEHD
 22   425994.1987    0.0004   425994.198  0.00231    0.015 0.97     0.97
 23    33716.5441    0.0384    33716.506  0.01012    1.136 0.95     0.96
 24    29207.1454   -0.0225    29207.168  0.00885   -0.790 0.95     0.96
Vec09: MIL1 -> BEHD
 25   122923.6872    0.0007   122923.687  0.00231    0.083 0.87     0.92
 26   -59878.5719    0.0006   -59878.573  0.01012    0.028 0.91     0.94
 27   -71746.5240    0.0016   -71746.526  0.00885    0.078 0.90     0.94
Vec10: MBYC -> BEHD
 28   -15820.3673   -0.0036   -15820.364  0.00297   -0.666 0.81     0.85
 29   -49489.4261    0.0093   -49489.435  0.01284    0.426 0.78     0.90
 30   -52863.8515   -0.0035   -52863.848  0.01148   -0.176 0.83     0.91
Vec11: G317 -> MBYC
 31     3741.2373   -0.0004     3741.238  0.00284   -0.066 0.85     0.90
 32   -29439.1032   -0.0037   -29439.100  0.01222   -0.173 0.77     0.90
 33   -31821.5683    0.0044   -31821.573  0.01120    0.221 0.84     0.91
Vec12: SAG1 -> G317
 34  -189236.1416   -0.0021  -189236.139  0.00221   -0.178 0.89     0.92
 35   -52215.1569    0.0131   -52215.170  0.00955    0.564 0.95     0.94
 36   -37674.1254   -0.0139   -37674.112  0.00882   -0.658 0.88     0.94
Vec13: DET1 -> MBYC
 37  -257144.6734   -0.0099  -257144.663  0.00236   -0.657 0.89     0.89
 38    11588.8359   -0.0238    11588.860  0.01000   -0.931 0.92     0.94
 39    38736.8742    0.0212    38736.853  0.00910    0.902 0.93     0.94
Vec14: STB1 -> G317
 40    94703.1393   -0.0004    94703.140  0.00221   -0.066 0.85     0.88
 41  -120887.7727    0.0040  -120887.777  0.00955    0.179 0.89     0.94
 42  -130606.5306   -0.0157  -130606.515  0.00882   -0.688 0.94     0.94
Vec15: BEHD -> WLCI
 43   -46413.8842   -0.0001   -46413.884  0.00231   -0.014 0.90     0.94
 44   -99686.0721    0.0250   -99686.097  0.01012    0.890 0.96     0.97
 45  -109601.7420   -0.0166  -109601.725  0.00885   -0.679 0.94     0.96
Vec16: NLIB -> BEHD
 46   425994.2335    0.0352   425994.198  0.00231    1.398 0.96     0.95
 47    33716.5709    0.0652    33716.506  0.01012    1.976 0.94     0.95
 48    29207.1412   -0.0267    29207.168  0.00885   -0.995 0.93     0.95
Vec17: MBYC -> BEHD
 49   -15820.3763   -0.0126   -15820.364  0.00297   -2.325 0.80     0.83
 50   -49489.4339    0.0015   -49489.435  0.01284    0.074 0.74     0.86
 51   -52863.8539   -0.0059   -52863.848  0.01148   -0.326 0.80     0.88
Vec18: G317 -> MBYC
 52     3741.2415    0.0038     3741.238  0.00284    0.699 0.83     0.88
 53   -29439.1022   -0.0027   -29439.100  0.01222   -0.137 0.80     0.88
 54   -31821.5851   -0.0124   -31821.573  0.01120   -0.713 0.74     0.87
Vec19: SAG1 -> G317
 55  -189236.1477   -0.0082  -189236.139  0.00221   -0.686 0.92     0.93
 56   -52215.1632    0.0068   -52215.170  0.00955    0.283 0.94     0.94
 57   -37674.1318   -0.0203   -37674.112  0.00882   -0.962 0.89     0.94
Vec20: DET1 -> MBYC
 58  -257144.6977   -0.0342  -257144.663  0.00236   -2.159 0.93     0.93
 59    11588.8736    0.0139    11588.860  0.01000    0.524 0.92     0.93
 60    38736.8391   -0.0139    38736.853  0.00910   -0.616 0.91     0.93
Vec21: STB1 -> G317
 61    94703.1446    0.0049    94703.140  0.00221    0.648 0.89     0.90
 62  -120887.7869   -0.0102  -120887.777  0.00955   -0.472 0.90     0.93
```

```
 63  -130606.5361   -0.0212  -130606.515  0.00882  -0.966 0.91    0.93
Vec22: BEHD -> WLCI
 64   -46413.8783    0.0058   -46413.884  0.00231   0.864 0.91    0.93
 65   -99686.1022   -0.0051   -99686.097  0.01012  -0.183 0.94    0.94
 66  -109601.7399   -0.0145  -109601.725  0.00885  -0.614 0.91    0.94

Sum of traditional redundancy numbers   =   57.00
Sum of standardized reliability numbers =   59.86

Estimated baseline outliers and minimum detectible outliers in meters
alpha = 0.01, beta = 0.80, r1 = 3, r2 = 54, non-central param. = 8.74
F(0.01;3,54) = 4.17
No.from   to  est. outlier [dX,dY,dZ] T   min. detect.[dX,dY,dZ] Ex Rel
 1 MBYC->G317 [ 0.001,-0.014, 0.002] 0.68 [0.0101,-0.0069,0.0202] 1.518
 2 SAG1->G317 [-0.002, 0.022,-0.006] 2.05 [0.0097,-0.0062,0.0192] 0.894
 3 DET1->MBYC [ 0.005,-0.008, 0.003] 0.35 [0.0099,-0.0065,0.0193] 1.418
 4 BEHD->MBYC [-0.005,-0.006, 0.014] 0.81 [0.0102,-0.0072,0.0204] 1.503
 5 NLIB->BEHD [ 0.004, 0.023,-0.018] 0.45 [0.0096,-0.0076,0.0197] 1.005
 6 MIL1->BEHD [ 0.011,-0.033, 0.023] 3.39 [0.0086,-0.0062,0.0174] 1.446
 7 G317->STB1 [ 0.001, 0.006,-0.005] 0.08 [0.0090,-0.0059,0.0182] 1.186
 8 NLIB->BEHD [-0.007, 0.032,-0.021] 0.83 [0.0135,-0.0107,0.0278] 0.374
 9 MIL1->BEHD [ 0.001,-0.004, 0.006] 0.05 [0.0095,-0.0068,0.0190] 0.782
10 MBYC->BEHD [-0.004, 0.007,-0.000] 0.29 [0.0113,-0.0079,0.0224] 1.357
11 G317->MBYC [ 0.000, 0.014,-0.012] 0.33 [0.0110,-0.0075,0.0221] 0.991
12 SAG1->G317 [ 0.002, 0.022,-0.017] 0.59 [0.0108,-0.0069,0.0213] 0.602
13 DET1->MBYC [-0.010,-0.021, 0.020] 1.82 [0.0122,-0.0080,0.0237] 0.731
14 STB1->G317 [-0.002, 0.004,-0.012] 0.39 [0.0097,-0.0064,0.0197] 0.853
15 BEHD->WLCI [-0.001, 0.034,-0.016] 0.85 [0.0115,-0.0085,0.0228] 0.395
16 NLIB->BEHD [ 0.000, 0.048,-0.018] 2.56 [0.0142,-0.0112,0.0292] 0.350
17 MBYC->BEHD [-0.012, 0.001,-0.007] 1.26 [0.0118,-0.0083,0.0235] 1.527
18 G317->MBYC [ 0.005, 0.003,-0.016] 1.05 [0.0111,-0.0075,0.0221] 1.237
19 SAG1->G317 [ 0.013, 0.015,-0.019] 2.39 [0.0119,-0.0076,0.0235] 0.544
20 DET1->MBYC [-0.016, 0.012,-0.012] 1.28 [0.0125,-0.0082,0.0244] 0.624
21 STB1->G317 [-0.005,-0.011, 0.000] 0.78 [0.0107,-0.0071,0.0217] 0.692
22 BEHD->WLCI [ 0.008, 0.019,-0.011] 0.74 [0.0116,-0.0086,0.0230] 0.542
```

Appendix I

Weighted BLIMPBE for New Fiducial Points, 22 Observed Baseline Vectors, with $\bar{S} = \left(S + N\right)^{-1}$

```
GPS observation variances and covariances scaled by 48.0 beginning at
observation 1.
The 3x3 block diagonal covariance matrix is replaced by the full
(session) matrix.

Adjustment type: Weighted Best LInear Minimum Bias Estimation with the
first 6 points selected

No of observations          :   66
Rank of A                   : - 24
                               ----
System redundancy           :   42

Adjustment PASSED the Chi Square test at the 95% Confidence L
Lower bound:     25.999
Chi Sq stat:     42.708
Upper bound:     61.777

Centering errors:
Name   horiz [m] vert [m]
BEHD   0.003      0.000
G317   0.003      0.000
MBYC   0.003      0.000

Estimated parameters: Cartesian (meters):
Name         X              Y              Z
DET1    568024.7235   -4690674.6294   4270188.8016
MIL1    172135.9978   -4668696.6225   4327808.3219
NLIB   -130934.5109   -4762291.7456   4226854.6572
SAG1    496374.9568   -4597431.5239   4378421.3553
STB1    212435.6784   -4528758.8969   4471353.7495
WLCI    248645.8081   -4828261.2877   4146460.0706
BEHD    295059.6899   -4728575.2121   4256061.8087
G317    307138.8170   -4649646.6773   4340747.2321
MBYC    310880.0559   -4679085.7766   4308925.6589
```

```
Estimated parameters: geodetic (ddd.mmssssssss):
Name   latitude         longitude       height
DET1   42.1750454137  -83.0543066755   145.0252
MIL1   43.0009130899  -87.5318408963   147.3430
NLIB   41.4617727264  -91.3429618948   207.0445
SAG1   43.3743119504  -83.5015959105   149.2319
STB1   44.4743748125  -87.1851587806   148.8232
WLCI   40.4830269186  -87.0307149945   180.3856
BEHD   42.0731982723  -86.2545890480   156.0737
G317   43.0942930838  -86.1314659800   155.7239
MBYC   42.4614129597  -86.1155807693   143.2370

Trace of estimated dispersion matrix: 0.002171
Estimated reference variance: 1.0169

Estimated standard errors (scaled by sqrt estimated reference
Name     std(X)   std(Y)   std(Z)    std(n)  std(e)   std(up)
           m        m        m         m        m        m
DET1    0.0049   0.0116   0.0106   0.0032   0.0049   0.0154
MIL1    0.0030   0.0113   0.0104   0.0033   0.0030   0.0150
NLIB    0.0027   0.0073   0.0058   0.0032   0.0027   0.0088
SAG1    0.0049   0.0110   0.0102   0.0036   0.0049   0.0146
STB1    0.0037   0.0103   0.0099   0.0040   0.0037   0.0137
WLCI    0.0038   0.0157   0.0135   0.0046   0.0037   0.0202
BEHD    0.0026   0.0105   0.0095   0.0027   0.0026   0.0139
G317    0.0036   0.0106   0.0098   0.0031   0.0036   0.0141
MBYC    0.0031   0.0107   0.0098   0.0027   0.0031   0.0143

Observation Estimates
Obs#       From-To
Obs#    dX/dY/dZ      Obs.      Adjusted    Obs.       Stu.  Trad. Std.
          Obs.       Error        Obs.    Std. Dev.   Res.   Red # Rel #
Vec01: MBYC -> G317
  1    -3741.2376   0.0013    -3741.239   0.00309   0.317  0.64  0.65
  2    29439.0952  -0.0041    29439.099   0.01099  -0.337  0.57  0.60
  3    31821.5696  -0.0036    31821.573   0.01009  -0.322  0.57  0.60
Vec02: SAG1 -> G317
  4  -189236.1424  -0.0026  -189236.140   0.00333  -0.812  0.52  0.55
  5   -52215.1555  -0.0021   -52215.153   0.01130  -0.180  0.42  0.55
  6   -37674.1125   0.0107   -37674.123   0.01044   0.928  0.66  0.58
Vec03: DET1 -> MBYC
  7  -257144.6615   0.0061  -257144.668   0.00364   2.147  0.41  0.47
  8    11588.8525  -0.0003    11588.853   0.01230  -0.026  0.45  0.53
  9    38736.8600   0.0027    38736.857   0.01111   0.224  0.60  0.56
Vec04: BEHD -> MBYC
 10    15820.3572  -0.0088    15820.366   0.00308  -2.162  0.64  0.63
 11    49489.4463   0.0109    49489.435   0.01157   0.810  0.62  0.59
 12    52863.8501  -0.0001    52863.850   0.01039  -0.013  0.56  0.59
Vec0 NLIB -> BEHD
 13   425994.2056   0.0048   425994.201   0.00456   1.959  0.26  0.33
 14    33716.5182  -0.0153    33716.534   0.01532  -0.935  0.47  0.52
 15    29207.1549   0.0034    29207.151   0.01327   0.235  0.59  0.53
Vec06: MIL1 -> BEHD
 16   122923.6979   0.0058   122923.692   0.00315   2.188  0.43  0.44
 17   -59878.6067  -0.0171   -59878.590   0.01255  -1.514  0.41  0.42
```

```
 18   -71746.5038    0.0094   -71746.513   0.01116    0.959  0.37   0.41
Vec07: G317 -> STB1
 19   -94703.1397   -0.0011   -94703.139   0.00272   -0.355  0.57   0.58
 20   120887.7846    0.0042   120887.780   0.01090    0.351  0.55   0.56
 21   130606.5082   -0.0092   130606.517   0.01047   -0.804  0.54   0.56
Vec08: NLIB -> BEHD
 22   425994.1987   -0.0021   425994.201   0.00456   -0.096  0.86   0.71
 23    33716.5441    0.0106    33716.534   0.01532    0.383  0.74   0.73
 24    29207.1454   -0.0061    29207.151   0.01327   -0.263  0.75   0.73
Vec09: MIL1 -> BEHD
 25   122923.6872   -0.0049   122923.692   0.00315   -0.706  0.57   0.55
 26   -59878.5719    0.0177   -59878.590   0.01255    0.995  0.59   0.57
 27   -71746.5240   -0.0108   -71746.513   0.01116   -0.667  0.63   0.58
Vec10: MBYC -> BEHD
 28   -15820.3673   -0.0013   -15820.366   0.00308   -0.285  0.68   0.67
 29   -49489.4261    0.0093   -49489.435   0.01157    0.483  0.71   0.68
 30   -52863.8515   -0.0013   -52863.850   0.01039   -0.071  0.73   0.68
Vec11: G317 -> MBYC
 31     3741.2373   -0.0016     3741.239   0.00309   -0.347  0.68   0.67
 32   -29439.1032   -0.0039   -29439.099   0.01099   -0.208  0.70   0.68
 33   -31821.5683    0.0049   -31821.573   0.01009    0.273  0.77   0.70
Vec12: SAG1 -> G317
 34  -189236.1416   -0.0018  -189236.140   0.00333   -0.174  0.70   0.67
 35   -52215.1569   -0.0035   -52215.153   0.01130   -0.180  0.76   0.70
 36   -37674.1254   -0.0022   -37674.123   0.01044   -0.128  0.68   0.69
Vec13: DET1 -> MBYC
 37  -257144.6734   -0.0058  -257144.668   0.00364   -0.442  0.74   0.68
 38    11588.8359   -0.0169    11588.853   0.01230   -0.797  0.73   0.73
 39    38736.8742    0.0169    38736.857   0.01111    0.863  0.76   0.74
Vec14: STB1 -> G317
 40    94703.1393    0.0007    94703.139   0.00272    0.121  0.67   0.66
 41  -120887.7727    0.0077  -120887.780   0.01090    0.409  0.69   0.69
 42  -130606.5306   -0.0132  -130606.517   0.01047   -0.691  0.76   0.70
Vec15: BEHD -> WLCI
 43   -46413.8842   -0.0023   -46413.882   0.00381   -0.546  0.43   0.42
 44   -99686.0721    0.0036   -99686.076   0.01753    0.179  0.50   0.54
 45  -109601.7420   -0.0039  -109601.738   0.01469   -0.216  0.54   0.55
Vec16: NLIB -> BEHD
 46   425994.2335    0.0327   425994.201   0.00456    1.479  0.89   0.76
 47    33716.5709    0.0374    33716.534   0.01532    1.398  0.79   0.70
 48    29207.1412   -0.0103    29207.151   0.01327   -0.479  0.66   0.69
Vec17: MBYC -> BEHD
 49   -15820.3763   -0.0103   -15820.366   0.00308   -2.251  0.68   0.68
 50   -49489.4339    0.0015   -49489.435   0.01157    0.087  0.68   0.68
 51   -52863.8539   -0.0037   -52863.850   0.01039   -0.227  0.71   0.68
Vec18: G317 -> MBYC
 52     3741.2415    0.0026     3741.239   0.00309    0.571  0.68   0.68
 53   -29439.1022   -0.0029   -29439.099   0.01099   -0.170  0.73   0.70
 54   -31821.5851   -0.0119   -31821.573   0.01009   -0.773  0.66   0.68
Vec19: SAG1 -> G317
 55  -189236.1477   -0.0079  -189236.140   0.00333   -0.761  0.79   0.75
 56   -52215.1632   -0.0098   -52215.153   0.01130   -0.484  0.82   0.74
 57   -37674.1318   -0.0086   -37674.123   0.01044   -0.494  0.66   0.71
Vec20: DET1 -> MBYC
 58  -257144.6977   -0.0301  -257144.668   0.00364   -2.177  0.84   0.79
```

```
 59    11588.8736    0.0208    11588.853   0.01230   0.943  0.82  0.72
 60    38736.8391   -0.0182    38736.857   0.01111  -0.975  0.64  0.69
Vec21: STB1 -> G317
 61    94703.1446    0.0060    94703.139   0.00272   0.933  0.76  0.75
 62  -120887.7869   -0.0065  -120887.780   0.01090  -0.361  0.76  0.73
 63  -130606.5361   -0.0187  -130606.517   0.01047  -1.021  0.70  0.71
Vec22: BEHD -> WLCI
 64   -46413.8783    0.0036   -46413.882   0.00381   0.699  0.57  0.57
 65   -99686.1022   -0.0265   -99686.076   0.01753  -1.323  0.50  0.45
 66  -109601.7399   -0.0018  -109601.738   0.01469  -0.104  0.46  0.44

Sum of traditional redundancy numbers   =   42.00
Sum of standardized reliability numbers =   41.23

Estimated baseline outliers and minimum detectible outliers in meters
alpha = 0.01, beta = 0.80, r1 = 3, r2 = 39, non-central param. = 8.90
F(0.01;3,39) = 4.33
No.from   to  est. outlier [dX,dY,dZ] T   min. detect.[dX,dY,dZ] Ex Rel
 1 MBYC->G317 [ 0.003,-0.002,-0.012] 0.98 [0.0121,-0.0083,0.0242] 5.836
 2 SAG1->G317 [-0.007, 0.002, 0.013] 1.86 [0.0123,-0.0079,0.0244] 6.578
 3 DET1->MBYC [ 0.017,-0.003, 0.000] 2.86 [0.0127,-0.0083,0.0248] 7.843
 4 BEHD->MBYC [-0.012,-0.000, 0.007] 1.61 [0.0125,-0.0088,0.0250] 6.494
 5 NLIB->BEHD [ 0.008,-0.010,-0.004] 1.06 [0.0147,-0.0116,0.0302]13.995
 6 MIL1->BEHD [ 0.009,-0.024, 0.014] 1.14 [0.0123,-0.0088,0.0247]11.656
 7 G317->STB1 [-0.002, 0.005,-0.012] 0.37 [0.0110,-0.0073,0.0223] 5.996
 8 NLIB->BEHD [-0.008,-0.003, 0.001] 0.27 [0.0159,-0.0126,0.0326] 3.643
 9 MIL1->BEHD [-0.009, 0.024,-0.014] 1.14 [0.0123,-0.0088,0.0247] 7.116
10 MBYC->BEHD [-0.001,-0.002, 0.012] 0.45 [0.0129,-0.0090,0.0257] 4.315
11 G317->MBYC [-0.001, 0.003,-0.001] 0.02 [0.0127,-0.0087,0.0254] 4.041
12 SAG1->G317 [-0.002,-0.002,-0.002] 0.10 [0.0129,-0.0082,0.0255] 4.405
13 DET1->MBYC [-0.009,-0.017, 0.020] 1.32 [0.0140,-0.0092,0.0273] 3.646
14 STB1->G317 [ 0.001, 0.009,-0.013] 0.16 [0.0115,-0.0076,0.0232] 4.476
15 BEHD->WLCI [-0.009, 0.018,-0.008] 0.61 [0.0163,-0.0121,0.0322] 9.379
16 NLIB->BEHD [-0.002, 0.019, 0.002] 1.18 [0.0168,-0.0133,0.0344] 3.739
17 MBYC->BEHD [-0.012, 0.002,-0.005] 1.19 [0.0133,-0.0093,0.0265] 4.152
18 G317->MBYC [ 0.004,-0.005,-0.012] 1.27 [0.0127,-0.0086,0.0253] 4.129
19 SAG1->G317 [ 0.013,-0.006,-0.008] 2.23 [0.0136,-0.0087,0.0270] 3.365
20 DET1->MBYC [-0.014, 0.026,-0.023] 1.06 [0.0144,-0.0094,0.0281] 3.471
21 STB1->G317 [-0.004,-0.003,-0.002] 0.25 [0.0122,-0.0080,0.0247] 3.346
22 BEHD->WLCI [ 0.009,-0.018, 0.008] 0.61 [0.0163,-0.0121,0.0322] 9.250
```

APPENDIX J

SCLESS for New Fiducial Points, 22 Observed Baseline Vectors

```
GPS observation variances and covariances scaled by 48.0 beginning at
observation 1.
The 3x3 block diagonal covariance matrix is replaced by a full
(session) matrix.

Adjustment type: Stochastically Constrained Least-Squares Solution

No of observations                      :   66
No. parameters                          : - 27
Rank of K                               : + 18
                                           ----
System redundancy                       :   57

Adjustment PASSED the Chi Square test at the 95% Confidence Level
Lower bound:     38.027
Chi Sq stat:     56.251
Upper bound:     79.752

Centering errors:
Name   horiz [m] vert [m]
BEHD   0.003      0.000
G317   0.003      0.000
MBYC   0.003      0.000

Estimated parameters: Cartesian (meters):
Name         X              Y              Z
DET1    568024.7202   -4690674.6421   4270188.8126
MIL1    172135.9992   -4668696.6415   4327808.3392
NLIB   -130934.5088   -4762291.7309   4226854.6491
SAG1    496374.9537   -4597431.5247   4378421.3557
STB1    212435.6774   -4528758.9069   4471353.7559
WLCI    248645.8074   -4828261.3126   4146460.0919
BEHD    295059.6893   -4728575.2206   4256061.8177
G317    307138.8157   -4649646.6859   4340747.2395
MBYC    310880.0544   -4679085.7852   4308925.6668

Estimated parameters: geodetic (ddd.mmsssssss):
Name   latitude        longitude       height
DET1   42.1750454135  -83.0543066963  145.0416
MIL1   43.0009130887  -87.5318408933  147.3687
NLIB   41.4617727387  -91.3429618874  207.0281
SAG1   43.3743119503  -83.5015959245  149.2326
```

```
STB1   44.4743748044  -87.1851587873  148.8348
WLCI   40.4830269184  -87.0307150029  180.4183
BEHD   42.0731982756  -86.2545890528  156.0860
G317   43.0942930824  -86.1314659883  155.7352
MBYC   42.4614129598  -86.1155807783  143.2485

Trace of estimated dispersion matrix: 0.000981
Estimated reference variance:  0.9869

Estimated standard errors (scaled by sqrt estimated reference variance)
Name     std(X)   std(Y)   std(Z)    std(n)  std(e)   std(up)
           m        m        m         m        m        m
DET1    0.0034   0.0065   0.0060   0.0032   0.0033   0.0082
MIL1    0.0030   0.0065   0.0061   0.0033   0.0030   0.0083
NLIB    0.0021   0.0038   0.0034   0.0030   0.0021   0.0041
SAG1    0.0033   0.0062   0.0060   0.0033   0.0033   0.0080
STB1    0.0031   0.0061   0.0060   0.0033   0.0031   0.0079
WLCI    0.0033   0.0073   0.0064   0.0036   0.0033   0.0090
BEHD    0.0028   0.0095   0.0084   0.0032   0.0027   0.0122
G317    0.0031   0.0091   0.0084   0.0033   0.0030   0.0119
MBYC    0.0029   0.0094   0.0086   0.0033   0.0028   0.0123

Observation Estimates
Obs#       From-To
Obs#    dX/dY/dZ       Obs.      Adjusted    Obs.       Stu.  Trad. Std.
          Obs.        Error        Obs.    Std. Dev.   Res.  Red # Rel #
Vec01: MBYC -> G317
  1    -3741.2376    0.0011     -3741.239  0.00285   0.269  0.71  0.73
  2    29439.0952   -0.0040     29439.099  0.01079  -0.339  0.61  0.72
  3    31821.5696   -0.0031     31821.573  0.00990  -0.284  0.62  0.72
Vec02: SAG1 -> G317
  4  -189236.1424   -0.0044   -189236.138  0.00277  -1.219  0.63  0.63
  5   -52215.1555    0.0057    -52215.161  0.00966   0.451  0.61  0.65
  6   -37674.1125    0.0037    -37674.116  0.00894   0.294  0.75  0.66
Vec03: DET1 -> MBYC
  7  -257144.6615    0.0043   -257144.666  0.00289   1.224  0.58  0.59
  8    11588.8525   -0.0044     11588.857  0.01018  -0.316  0.66  0.65
  9    38736.8600    0.0058     38736.854  0.00928   0.446  0.71  0.66
Vec04: BEHD -> MBYC
 10    15820.3572   -0.0079     15820.365  0.00283  -1.899  0.72  0.74
 11    49489.4463    0.0109     49489.435  0.01133   0.823  0.67  0.73
 12    52863.8501    0.0010     52863.849  0.01016   0.088  0.63  0.72
Vec05: NLIB -> BEHD
 13   425994.2056    0.0074    425994.198  0.00296   1.799  0.67  0.67
 14    33716.5182    0.0079     33716.510  0.00980   0.397  0.79  0.68
 15    29207.1549   -0.0137     29207.169  0.00860  -0.789  0.76  0.67
Vec06: MIL1 -> BEHD
 16   122923.6979    0.0078    122923.690  0.00280   2.647  0.53  0.55
 17   -59878.6067   -0.0277    -59878.579  0.01032  -2.123  0.64  0.58
 18   -71746.5038    0.0177    -71746.521  0.00913   1.548  0.58  0.57
Vec07: G317 -> STB1
 19   -94703.1397   -0.0014    -94703.138  0.00249  -0.432  0.63  0.64
 20   120887.7846    0.0056    120887.779  0.00950   0.438  0.69  0.66
 21   130606.5082   -0.0082    130606.516  0.00898  -0.662  0.69  0.67
```

```
Vec08: NLIB -> BEHD
 22   425994.1987    0.0005    425994.198  0.00296   0.025  0.91  0.80
 23    33716.5441    0.0338     33716.510  0.00980   1.147  0.88  0.81
 24    29207.1454   -0.0232     29207.169  0.00860  -0.936  0.87  0.81
Vec09: MIL1 -> BEHD
 25   122923.6872   -0.0029    122923.690  0.00280  -0.422  0.65  0.65
 26   -59878.5719    0.0071    -59878.579  0.01032   0.378  0.75  0.69
 27   -71746.5240   -0.0025    -71746.521  0.00913  -0.148  0.75  0.69
Vec10: MBYC -> BEHD
 28   -15820.3673   -0.0022    -15820.365  0.00283  -0.476  0.74  0.75
 29   -49489.4261    0.0093    -49489.435  0.01133   0.486  0.75  0.79
 30   -52863.8515   -0.0024    -52863.849  0.01016  -0.139  0.78  0.79
Vec11: G317 -> MBYC
 31     3741.2373   -0.0014      3741.239  0.00285  -0.306  0.74  0.76
 32   -29439.1032   -0.0040    -29439.099  0.01079  -0.212  0.73  0.78
 33   -31821.5683    0.0044    -31821.573  0.00990   0.251  0.80  0.79
Vec12: SAG1 -> G317
 34  -189236.1416   -0.0036   -189236.138  0.00277  -0.347  0.74  0.72
 35   -52215.1569    0.0043    -52215.161  0.00966   0.218  0.84  0.76
 36   -37674.1254   -0.0092    -37674.116  0.00894  -0.515  0.76  0.76
Vec13: DET1 -> MBYC
 37  -257144.6734   -0.0076   -257144.666  0.00289  -0.580  0.78  0.74
 38    11588.8359   -0.0210     11588.857  0.01018  -0.959  0.83  0.79
 39    38736.8742    0.0200     38736.854  0.00928   0.996  0.83  0.80
Vec14: STB1 -> G317
 40    94703.1393    0.0010     94703.138  0.00249   0.171  0.71  0.71
 41  -120887.7727    0.0063   -120887.779  0.00950   0.327  0.78  0.75
 42  -130606.5306   -0.0142   -130606.516  0.00898  -0.730  0.83  0.76
Vec15: BEHD -> WLCI
 43   -46413.8842   -0.0022    -46413.882  0.00326  -0.478  0.59  0.59
 44   -99686.0721    0.0200    -99686.092  0.01130   0.843  0.81  0.73
 45  -109601.7420   -0.0162   -109601.726  0.00981  -0.785  0.76  0.72
Vec16: NLIB -> BEHD
 46   425994.2335    0.0353    425994.198  0.00296   1.605  0.93  0.84
 47    33716.5709    0.0606     33716.510  0.00980   2.109  0.88  0.80
 48    29207.1412   -0.0274     29207.169  0.00860  -1.174  0.85  0.79
Vec17: MBYC -> BEHD
 49   -15820.3763   -0.0112    -15820.365  0.00283  -2.415  0.74  0.75
 50   -49489.4339    0.0015    -49489.435  0.01133   0.085  0.71  0.77
 51   -52863.8539   -0.0048    -52863.849  0.01016  -0.304  0.76  0.78
Vec18: G317 -> MBYC
 52     3741.2415    0.0028      3741.239  0.00285   0.601  0.74  0.76
 53   -29439.1022   -0.0030    -29439.099  0.01079  -0.174  0.76  0.78
 54   -31821.5851   -0.0124    -31821.573  0.00990  -0.814  0.70  0.76
Vec19: SAG1 -> G317
 55  -189236.1477   -0.0097   -189236.138  0.00277  -0.934  0.81  0.79
 56   -52215.1632   -0.0020    -52215.161  0.00966  -0.095  0.86  0.79
 57   -37674.1318   -0.0156    -37674.116  0.00894  -0.871  0.76  0.78
Vec20: DET1 -> MBYC
 58  -257144.6977   -0.0319   -257144.666  0.00289  -2.314  0.86  0.83
 59    11588.8736    0.0167     11588.857  0.01018   0.735  0.86  0.78
 60    38736.8391   -0.0151     38736.854  0.00928  -0.780  0.77  0.77
Vec21: STB1 -> G317
 61    94703.1446    0.0063     94703.138  0.00249   0.981  0.79  0.79
```

```
 62  -120887.7869   -0.0079   -120887.779  0.00950  -0.429  0.82  0.78
 63  -130606.5361   -0.0197   -130606.516  0.00898  -1.053  0.79  0.77
Vec22: BEHD -> WLCI
 64   -46413.8783    0.0037    -46413.882  0.00326   0.689  0.69  0.70
 65   -99686.1022   -0.0101    -99686.092  0.01130  -0.428  0.79  0.67
 66  -109601.7399   -0.0141   -109601.726  0.00981  -0.710  0.71  0.65

Sum of traditional redundancy numbers   =   57.00
Sum of standardized reliability numbers =   56.64

Estimated baseline outliers and minimum detectable outliers in meters
alpha = 0.01, beta = 0.80, r1 = 3, r2 = 54, non-central param. = 8.90
F(0.01;3,54) = 4.17
No. from  to  est. outlier [dX,dY,dZ] T   min. detect.[dX,dY,dZ] Ex Rel
 1 MBYC->G317 [ 0.003,-0.009,-0.004] 0.75 [0.0111,-0.0075,0.0221] 3.413
 2 SAG1->G317 [-0.007, 0.014, 0.001] 1.96 [0.0114,-0.0072,0.0225] 4.230
 3 DET1->MBYC [ 0.011,-0.006, 0.003] 1.43 [0.0115,-0.0076,0.0225] 4.855
 4 BEHD->MBYC [-0.008,-0.003, 0.010] 1.22 [0.0111,-0.0078,0.0221] 3.223
 5 NLIB->BEHD [ 0.007, 0.019,-0.022] 0.91 [0.0112,-0.0089,0.0230] 4.381
 6 MIL1->BEHD [ 0.010,-0.031, 0.021] 2.51 [0.0107,-0.0077,0.0215] 6.700
 7 G317->STB1 [-0.002, 0.004,-0.008] 0.19 [0.0104,-0.0068,0.0210] 4.336
 8 NLIB->BEHD [-0.007, 0.025,-0.020] 0.62 [0.0149,-0.0118,0.0306] 2.161
 9 MIL1->BEHD [-0.005, 0.004, 0.002] 0.39 [0.0112,-0.0081,0.0226] 4.525
10 MBYC->BEHD [-0.002, 0.004, 0.004] 0.31 [0.0122,-0.0085,0.0242] 2.822
11 G317->MBYC [-0.001, 0.009,-0.007] 0.15 [0.0120,-0.0082,0.0240] 2.620
12 SAG1->G317 [-0.002, 0.012,-0.013] 0.18 [0.0123,-0.0078,0.0243] 3.193
13 DET1->MBYC [-0.009,-0.018, 0.020] 1.54 [0.0134,-0.0088,0.0262] 2.633
14 STB1->G317 [ 0.000, 0.007,-0.011] 0.17 [0.0110,-0.0073,0.0223] 3.387
15 BEHD->WLCI [-0.005, 0.032,-0.018] 0.87 [0.0140,-0.0104,0.0277] 4.564
16 NLIB->BEHD [-0.000, 0.043,-0.018] 2.11 [0.0156,-0.0123,0.0320] 2.038
17 MBYC->BEHD [-0.012, 0.002,-0.006] 1.36 [0.0126,-0.0088,0.0250] 2.776
18 G317->MBYC [ 0.004,-0.001,-0.014] 1.31 [0.0120,-0.0082,0.0240] 2.782
19 SAG1->G317 [ 0.012, 0.005,-0.016] 2.44 [0.0131,-0.0084,0.0260] 2.483
20 DET1->MBYC [-0.014, 0.018,-0.016] 1.05 [0.0137,-0.0090,0.0268] 2.378
21 STB1->G317 [-0.004,-0.007, 0.001] 0.40 [0.0117,-0.0077,0.0238] 2.450
22 BEHD->WLCI [ 0.008, 0.014,-0.012] 0.60 [0.0138,-0.0103,0.0274] 4.265
```